Introduction

Much is known about measuring mortality inequality within populations; one example is e† (e-dagger) - the average life expectancy lost due to death. Less is known about measuring health inequalities. Existing strategies fall into two groups: prevalence-based and incidence-based. The first group [1–4], tracing back to Sullivan's method, focuses on outcomes rather than the process of becoming unhealthy. Prevalence data capture states, not transitions, and makes no distinction between reversible and irreversible conditions. The second group uses transition intensities from longitudinal data, accounting for individual transitions, distinguishing between reversible and irreversible states, and not assuming independence between survival and disability. To our knowledge, only three studies have used this approach: [5, 6] and our recent contribution [7], where we proposed a new distance-based measure of health inequality. These indices, however, were not decomposed, though it is theoretically possible, and lacked clear demographic motivation.

Drawing inspiration from [7] and [1], this article advances this research by introducing a novel decomposition method for multistate healthy and unhealthy life expectancy - a multistate extension of e† that accounts for life years lost due to death and those lost or gained through non-lethal transitions, for any initial state. We aim for this index to be decomposable, interpretable, and demographically motivated.

Data and methods

We use data from the Survey of Health, Ageing and Retirement in Europe (SHARE) to obtain individuals' sex, age, health status at time t and $t + 1$, and calendar period. While we focus on self-reported health in the results, we also compute our health inequality index using four alternative measures: GALI, ADL, IADL, and chronic conditions. Using all available SHARE waves, we restrict the analysis to Spain. The model includes three states: Healthy (h), Unhealthy (u), and Dead (d), with recoveries allowed. States h and u are transient, and d is absorbing, yielding six possible transitions. The state space diagram for a given combination of sex, period, and age is shown in Figure 1.

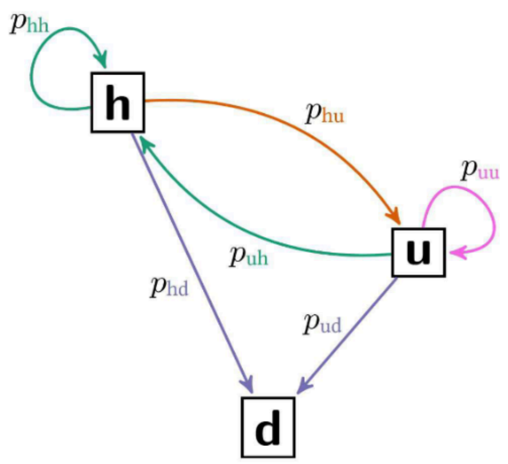

Figure 1: State space diagram

Transition probabilities are estimated with the msm package [8], using exact ages and transition times, with death dates treated as known. Separate sex-specific models produce age-specific transition matrices (ages 20–110). We then adjust p^{ud}, and p^{hd} using mortality data from the HMD [9] and estimated disability prevalence. The resulting probabilities are used to build a multi-state life table via the fundamental matrix $N = (I - U)^{-1}$, from which survival curves and expected time in each state are derived following [1, 6]. Corresponding transfers can be obtained by multiplying the transition probability by survival in the initial state, such as:

$$t^{hd}(x) = p^{hd}(x) \cdot l^{h}(x),$$
$$t^{hu}(x) = p^{hu}(x) \cdot l^{h}(x),$$
$$t^{uh}(x) = p^{uh}(x) \cdot l^{u}(x),$$
$$t^{ud}(x) = p^{ud}(x) \cdot l^{u}(x).$$

Using these equations for transfers, we can calculate all necessary multistate life table quantities *(d(x), l(x), e(x), and e†) using* standard demographic techniques. Given a set of transition probabilities $p^{hh}(x)$, $p^{hu}(x)$, $p^{hd}(x)$, $p^{uu}(x)$, $p^{uh}(x)$, $p^{ud}(x)$ and survival curves $l^{ij}(x)$, we obtain four average times spent in each state due to transitions to h or u: $HLE^{h}(x)$, $HLE^{u}(x)$, $ULE^{h}(x)$, $ULE^{u}(x)$, which can be further summarised to HLE and ULE. Using these expectancies and transfers, we calculate the decomposed HLE† and ULE†. Each component of LE† can be expressed as four elements, some positive and some negative, reflecting losses or gains from transitions, which sum to give the total LE† value. Thus:

$$HLE_{\dagger}^{hu}(x) = \sum_{x}^{\omega} \overbrace{t^{hu}(x)}^{\text{number loosing health}} \cdot \overbrace{(1 + HLE^{h}(x+1) - HLE^{u}(x+1))}^{\text{HLE loss due to transition}},$$

$$HLE_{\dagger}^{hd}(x) = \sum_{x}^{\omega} t^{hd}(x) \cdot HLE^{h}(x),$$

$$HLE_{\dagger}^{uh}(x) = \sum_{x}^{\omega} t^{uh}(x) \cdot (-1 + HLE^{u}(x+1) - HLE^{h}(x+1)),$$

$$HLE_{\dagger}^{ud}(x) = \sum_{x}^{\omega} t^{ud}(x) \cdot HLE^{u}(x),$$

$$ULE_{\dagger}^{hu}(x) = \sum_{x}^{\omega} t^{hu}(x) \cdot (-1 + ULE^{h}(x+1) - ULE^{u}(x+1)),$$

$$ULE_{\dagger}^{hd}(x) = \sum_{x}^{\omega} t^{hd}(x) \cdot ULE^{h}(x),$$

$$ULE_{\dagger}^{uh}(x) = \sum_{x}^{\omega} t^{uh}(x) \cdot (1 + ULE^{u}(x+1) - ULE^{h}(x+1)),$$

$$ULE_{\dagger}^{ud}(x) = \sum_{x}^{\omega} t^{ud}(x) \cdot ULE^{u}(x),$$

$$\begin{aligned} LE_{\dagger}^{hu}(x) &= HLE_{\dagger}^{hu} + ULE_{\dagger}^{hu}, \\ LE_{\dagger}^{hd}(x) &= HLE_{\dagger}^{hd} + ULE_{\dagger}^{hd}, \\ LE_{\dagger}^{uh}(x) &= HLE_{\dagger}^{uh} + ULE_{\dagger}^{uh}, \\ LE_{\dagger}^{ud}(x) &= HLE_{\dagger}^{ud} + ULE_{\dagger}^{ud}. \end{aligned}$$

Adding the elements of the final four equations gives their contributions to LE†. Specifically, $LE_{hd}^{\dagger}(x) + LE_{ud}^{\dagger}(x)$ gives the mortality component that is > 0, while summing all four elements yields the complete MSLT index, including those from non-lethal transitions that can be either positive or negative. Summing the four values within the first two groups yields HLE† and ULE†; adding them yields LE† = HLE† + ULE†.

Some Results

We present 2017 results for self-perceived health to illustrate key features of our approach. Results for other health measures and years can be reproduced using the provided code. Figure 2 shows MSLT† components in years. Contributions are divided into healthy and unhealthy groups, each with four transitions. Becoming healthy reduces MSLT†, while other transitions increase it; for the unhealthy group, becoming unhealthy reduces it. Non-lethal transitions contribute more individually but partially cancel, making the net effect of death transitions larger. Overall, MSLT† is higher in males than in females.

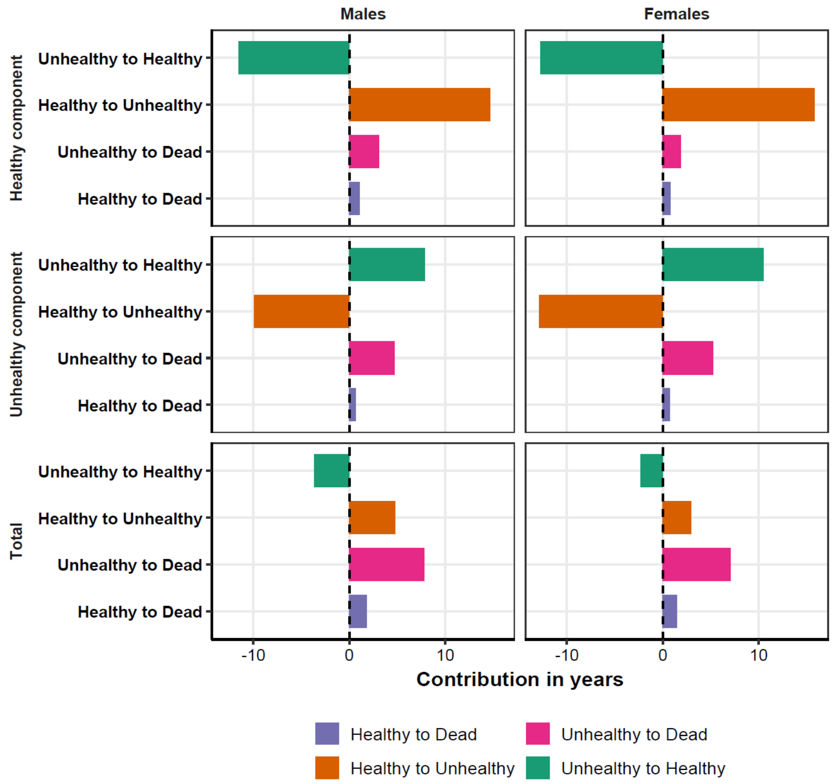

Figure 2: Components of MSLT† (HLE†, ULE†, and Total†)

Figure 3 compares classic e† with MSLT† decomposed by transition-specific contributions. Both are higher in males than in females. The mortality component of MSLT† is 9.58 years (males) and 8.56 years (females), slightly below e† (10.09 and 8.77 years, respectively). Including non-lethal transitions (1.09 and 0.627 years) gives the full MSLT†, exceeding e†.

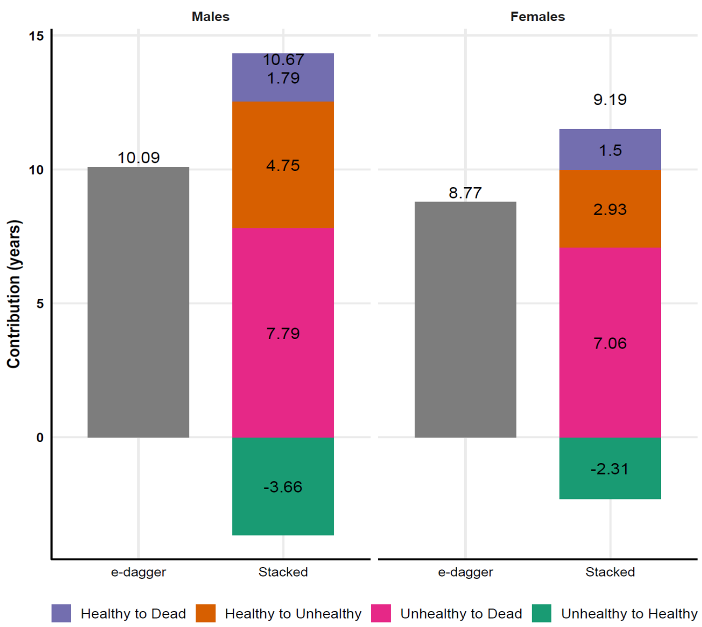

Figure 3: Transition-specific contribution to MSLT† in comparison with e†

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