The contribution of forecast uncertainty to lifespan uncertainty

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Abstract

Background
Period lifespan variability is sometimes interpreted as an indicator of lifespan uncertainty for individual cohort members. Is this an admissible interpretation for living cohort members, given that future death rates are not precisely known?

Objective
To answer this question, we express individual lifespan uncertainty as a function of variation in ages at death in a cohort lifetable and uncertainty about future death rates. We then quantify the importance of each component to total individual lifespan variability of a birth cohort. A small contribution of the forecast variance would support the interpretation of lifespan variability as individual lifespan uncertainty.

Methods
Using HMD data for seven European countries, we decompose the total variance in the remaining lifespan at birth, age 40, and 80 of an individual cohort member into cohort lifetable variance and forecast variance for cohorts born 1871-2019. We do so under Lee-Carter cohort forecast variance and under an empirical forecast variance measure which allows for mortality shocks.

Results
Even when accounting for the possibility of future mortality shocks, the forecast variance component tends to contribute less than 10% to the total lifespan variance of an individual. Exceptions are France and the small and thus volatile population of Iceland. Conditioning on the absence of future mortality shocks, the forecast variance contributes around 1% to total lifespan variance.

Contribution
We give credibility to the interpretation of period lifetable lifespan variance as individual lifespan uncertainty by showing that the lifespan variance of a cohort is mainly determined by expected lifetable variance with smaller contributions from forecast variance.

Key words
Variance decomposition; cohort mortality; mortality forecasting; lifespan variation; lifespan inequality

1 Introduction

The variability in the age at death, that is lifespan inequality, is the focus of ongoing research (Caswell 2023; Aburto et al. 2020; Van Raalte et al. 2018; Colchero et al. 2016; Seligman et al. 2016; Vaupel et al. 2011; Edwards and Tuljapurkar 2005) including the development of new measures (Nepomuceno et al. 2022), a focus on regional (Wilson et al. 2020) and socio-demographic disparities (Permanyer et al. 2018; Sasson 2016; Brown et al. 2012) and lifespan inequality in relation to economic decision making (Edwards 2013; Hurd et al. 2004). Like life expectancy, lifespan variability is estimated from either a cohort or a period lifetable and is a summary measure of the lifetable distribution of deaths.

Some research has interpreted lifetable lifespan variability as individual uncertainty (or predictability) of a person's eventual age at death (Nepomuceno et al. 2022; Van Raalte et al. 2018; Sasson 2016; Edwards 2013; Edwards and Tuljapurkar 2005). However, such a personal interpretation is contentious as the individual has access to different information about their future lifespan than is given by the lifetable alone. We distinguish three sources of information about individual lifespan: what the individual knows about themselves, what they know about others, and what they know about the future.

1. Personal information: Different strata of a population experience different levels of lifespan uncertainty. This has for example been shown across socioeconomic groups (Permanyer et al. 2018; Sasson 2016; Brown et al. 2012). Individual-level information on e.g., education, income, lifestyle factors and medical histories can reduce the individual lifespan uncertainty. For example, a life-time smoker knows about the detrimental effects of their health behaviour on their expected length of life. Therefore, the individual's lifespan uncertainty is not the same as the lifespan uncertainty researchers can estimate with limited information. However, Caswell (2023) shows that even with extreme levels of heterogeneity in the population, only a few percent of variance in the length of life can be attributed to inequality between groups. Instead, it is the within-lifetable variance that explains most of the lifespan variability.

2. Interpersonal information: Another source of individual uncertainty or certainty about the lifespan is the mortality experience of others. While the period lifetable is a source of such interpersonal information on lifespans, it is only a snapshot at a single point in time. Nepomuceno et al. (2022), argue that individuals base their assessment of their lifetime uncertainty not only on the current mortality experience of others (period mortality), but also on the past mortality experience of contemporary cohorts. I.e., "[...] it is possible that people adjust their survival expectations on the basis of the survival histories of a broader mixture of family members, colleagues, and neighbors [...] who were from mixed birth cohorts" (Nepomuceno et al. 2022, p188). Thus, their proposed cross-sectional measure of lifespan variability $CAL^{\dagger}$ takes both the period and the cohort perspective into account to allow for more informative monitoring of mortality changes.

3. Information about the future: Further, an individual has uncertainty about which death rates will apply to them. Only in a population in which age-specific death rates do not change with time would they know what their future mortality rates will look like. But mortality rates are subject to constant fluctuations and sudden mortality shocks, for example, due to pandemics, other natural disasters, and wars (Schöley et al. 2022). Additionally, it is uncertain to what degree long-term mortality trends will continue into the future as

illustrated by slowing rates of mortality improvements in some countries (Hum et al. 2015; Ford et al. 2019; Murphy 2021). Thus, an individual has uncertainty about future death rates, that is, how their lifetable will be completed. This "forecast uncertainty" adds to the individual's total lifespan uncertainty.

We focus on the contribution of forecast uncertainty to total lifespan uncertainty and ask whether the interpretation of lifespan variability as individual lifespan uncertainty is admissible for living members of a cohort. A small contribution of the forecast variance to the total lifespan variance would support the interpretation of lifespan variability as individual lifespan uncertainty.

To separate the uncertainty faced by an individual about the length of its lifespan we divide it into the lifetable component of variability, that is uncertainty *due to* the shape of the survival curve (lifetable lifespan variance), and the forecast component of variability, that is uncertainty *about* the shape of the survival curve (lifetable forecast variance), as the cohort lifetable has to be completed using forecasts of age-specific mortality rates. For an individual cohort member, due to the law of total variance:

$$\begin{aligned}&\text{Total Lifespan Variance} = \\ &\text{Expected Lifetable Lifespan Variance} + \text{Variance of Lifetable Forecasts.}\end{aligned}$$

We estimate each of the above components across a range of birth cohorts in seven countries. First, we perform stochastic Lee-Carter (Lee and Carter 1992) lifetable forecasts across birth cohorts. Adopting a methodology proposed by Caswell (2023), we then perform a variance decomposition of the total cohort lifespan variance into a lifetable variance and a forecast variance component. This allows us to examine the share of each component, and observe their changes and the development of the total lifespan variance over time.

We obtain results for the variance decomposition that relate to two different scenarios: A) We expect a linear future without mortality shocks or trend changes. In this scenario, we exclude the mortality data of the two world wars from the Lee-Carter model estimation. Because the uncertainty estimated via the original Lee-Carter model is known to be overly optimistic (Lee and Miller 2001; Booth et al. 2006; Li et al. 2009) and because we excluded the war years, we look at a second scenario B): a future with mortality shocks and trend changes. For this scenario, we use an empirical measure of forecast variance from a cross-validation exercise to estimate the forecast variance component – the Mean Square Error (MSE). The MSE is based on observed forecast errors. Thus, using this empirical measure, overly optimistic variance estimates and increased uncertainty due to extreme mortality events or trend changes are accounted for.

2 Data and methods

2.1 Data

We use data from the open-access Human Mortality Database (Max Planck Institute for Demographic Research, University of California, Berkeley, and French Institute for Demographic Studies 2023, HMD) for our analyses. The HMD provides high-quality population

and mortality data. We source male death counts and population exposures by single age for Denmark, England and Wales, France, Iceland, the Netherlands, Norway, and Sweden, for the years 1871 through 2019. The countries were chosen based on long available data series that are necessary for both the cross-validation of the cohort mortality forecasts and the desire for the analysis of multiple cohorts. Further, we perform the variance decomposition for forecasts of remaining life expectancy at different ages (age 0, age 40, and age 80), thus increasing the demand for longer data series. We have chosen to perform our analysis on male mortality to estimate the upper bound of lifespan uncertainty. Male death counts have historically been more volatile due to higher male participation in military conflicts. This volatility increases both the lifetable lifespan variance and the lifetable forecast variance. Similarly, we have included Denmark and Iceland whose mortality trends are more volatile due to smaller population sizes.

Cohort forecast specification

We forecast age specific mortality rates for birth cohorts 1901 through 2020 given the data available during the 30 year period preceding a cohort. Thus, the fitting window $p = 1$ corresponding to birth cohort $c = 1901$ is 1871 through 1900 and the fitting window $p = 120$ informing forecasts for birth cohort $p = 2019$ is 1990 through 2019.

Model specification: We fit a Poisson-Lee-Carter model where death counts at single age x and single year t are distributed $D_{x,t} \sim \text{Pois}(\mu_{x,t}\ E_{x,t})$, with $E_{x,t}$ being the person-years of exposure and $\mu_{x,t}$ the death rates. We use the original Lee-Carter specification $\mu_{x,t} = \exp(a_x + b_x k_t)$ with identifiability constraints $\sum_t k_t = 0$ and $\sum_x b_x = 1$.

Model fit: The model is fit via maximum-likelihood with quadratic penalties on the second differences of the age effects (ages zero to 100). These roughness penalties improve model convergence and ensure that forecasts for neighboring age groups are similar – an important consideration when deriving cohort life tables from period diagonals (Van Raalte et al. 2023). To further improve the stability of the forecasts, we penalize the squared difference between parameter estimates of two neighboring fitting periods, ensuring that forecasts for neighboring cohorts are similar. A ridge penalty on the age effects completes the set of penalties. The penalized log-likelihood function over parameters $\boldsymbol{\theta}^{(p)} = \{a_x, b_x, k_t : x \in \mathcal{X}, t \in \mathcal{T}^{(p)}\}$ for fitting period p then is

$$\begin{aligned}\ell(\boldsymbol{\theta}^{(p)}) = &\left[\sum_{x\in\mathcal{X}}\sum_{t\in\mathcal{T}^{(p)}} D_{x,t}\log\left(\mu_{x,t}^{(p)}\right) - E_{x,t}\mu_{x,t}^{(p)}\right] \\ &- \left(\lambda_1 p^{(p)}_{\text{a}_\text{x}\text{roughness}} + \lambda_2 p^{(p)}_{\text{b}_\text{x}\text{roughness}} + \lambda_3 p^{(p)}_{\text{deviation}} + \lambda_4 p^{(p)}_{\text{ridge}}\right), \end{aligned} \tag{1}$$

where

$$p^{(p)}_{\text{a}_\text{x}\text{roughness}} = \sum_{x=1}^{\omega-2}(a^{(p)}_{x+2} - 2a^{(p)}_{x+1} + a^{(p)}_{x})^2 \tag{2}$$

$$p^{(p)}_{\text{b}_\text{x}\text{roughness}} = \sum_{x=1}^{\omega-2}(b_{x+2} - 2b_{x+1} + b_{x})^2 \tag{3}$$

$$p^{(p)}_{\text{deviation}} = \sum_{p=2}^{30}\sum_{x=1}^{\omega}(a^{(p)}_{x} - a^{(p-1)}_{x})^2 + (b^{(p)}_{x} - b^{(p-1)}_{x})^2 \tag{4}$$

$$p^{(p)}_{\text{ridge}} = \sum_{x=1}^{\omega}(a^{(p)}_{x})^2 + (b^{(p)}_{x})^2. \tag{5}$$

Parameter uncertainty: We quantify parameter uncertainty via an approximation to the parametric bootstrap where we sample 2500 $\boldsymbol{\theta}^{(p)}$ parameter vectors from a multivariate Normal distribution with mean equal to the maximum likelihood parameter estimates and covariance equal to the inverse of the corresponding negative Hessian (King et al. 2000; Haberman and Renshaw 2009).

Model specification uncertainty: We chose a penalized Lee-Carter model to forecast the mortality based on extensive sensitivity checks. The results from these model comparisons on the influence of parameter uncertainty and on the importance of model specification can be found in the additional material.

Forecast: In order to forecast $\mu^{(p)}_{x,t}$ past the fitting period, we estimate the drift and the variance of the estimated year-on-year $k^{(p)}_t$ changes and simulate a corresponding random walk with drift over 100 years for each of the 2500 parameter draws. The resulting forecasts $\hat{k}^{(p)}_t$ for forecast horizon $t \in \mathcal{H}^{(p)}$ are then used to derive a corresponding surface of age-period mortality rates $\hat{\mu}^{(p)}_{x,t} = \exp\left(a^{(p)}_x + b^{(p)}_x \hat{k}^{(p)}_t\right)$.

We convert these period-age forecasts into cohort-age forecasts by extracting the corresponding cohort-diagonals. Let $J^{(p)}$ denote the jump-off year for the forecast corresponding to the fitting window p. For each period-age forecast $\hat{\mu}^{(p)}_{x,t}$ we then extract the forecast mortality rates for those cohorts being infants during the jump-off year, $\hat{\mu}^{(p)}_{x,t=J(p)+x}$, for those cohorts who turned 40 in the jump-off year, $\hat{\mu}^{(p)}_{x,t=J(p)-40+x}$, and for those cohorts who turned 80 in the jump-off year, $\hat{\mu}^{(p)}_{x,t=J(p)-80+x}$. Forecasts are done independently for each country. For a visual representation of the extraction of cohort-diagonals, see the additional material.

The years 1914–1921, and 1939–1945 have been excluded from the estimation of a_x and b_x, and from the k_t drift and variance estimates. This conditions the forecast variance on a future without extreme mortality shocks. For an estimate of the forecast variance allowing for the possibility of extreme shocks, we calculate the mean squared forecast error of our cohort life expectancy forecasts.

Cohort variance decomposition We denote by X the random age at death of a cohort member and with $\boldsymbol{\mu}$ the random vector of age-specific cohort death rates parameterizing the distribution of X. How much of the uncertainty about the individual's eventual lifespan is driven by within-lifetable stochasticity and how much is driven by the uncertainty about which future lifetable will apply to them? The law of total variance, as demonstrated in the context of lifetables by Caswell (2023), allows us to answer this question by decomposing the total variance in the individual's age at death, $\mathrm{Var}(X)$, into variance due to uncertainty in future death rates, $\mathrm{Var}_{\boldsymbol{\mu}}[\mathrm{E}(X|\boldsymbol{\mu})]$, and the expected variance inherent to any lifetable, $\mathrm{E}_{\boldsymbol{\mu}}[\mathrm{Var}(X|\boldsymbol{\mu})]$:

$$\underbrace{\mathrm{Var}(X)}_{\substack{\text{Total lifespan variance}}} = \underbrace{\mathrm{E}_{\boldsymbol{\mu}}[\mathrm{Var}(X|\boldsymbol{\mu})]}_{\substack{\text{Within-lifetable variance /}\\ \text{Expected lifespan variance}}} + \underbrace{\mathrm{Var}_{\boldsymbol{\mu}}[\mathrm{E}(X|\boldsymbol{\mu})]}_{\substack{\text{Between-lifetable variance /}\\ \text{Forecast variance}}} . \tag{6}$$

We employ the stochastic cohort lifetable forecasts $\boldsymbol{\mu}_s$ to decompose the total individual uncertainty about the remaining lifespan at birth. For brevity, we write $e_{0_s} = \mathrm{E}(X|\boldsymbol{\mu}_s)$ for the expected age at death (life expectancy) and $\sigma_s^2 = \mathrm{Var}(X|\boldsymbol{\mu}_s)$ for the variance in the age at death (lifespan variance) of a cohort member under mortality rates $\boldsymbol{\mu}_s$. Both quantities are calculated from stochastic cohort death rate forecasts $\boldsymbol{\mu}_s$ using the matrix lifetable equations in Caswell (2023).

For scenario A), a future without mortality shocks, we calculate the between-lifetable variance, that is the forecast variance of the cohort life expectancies, as

$$\mathrm{Var}_{\boldsymbol{\mu}}[\mathrm{E}(X|\boldsymbol{\mu})] = \frac{1}{500-1}\sum_s (e_{0_s} - \overline{e_0})^2, \tag{7}$$

where $\overline{e_0} = \frac{1}{500}\sum_s e_{0_s}$ is the mean cohort life expectancy forecast as averaged over the simulation runs.

The second part of the variance decomposition, the expected cohort lifespan variance, or within-lifetable variance, is calculated as the mean of the simulated lifespan variances:

$$\mathrm{E}_{\boldsymbol{\mu}}[\mathrm{Var}(X|\boldsymbol{\mu})] = \frac{1}{500}\sum_s \sigma_s^2. \tag{8}$$

Empirical forecast error quantification For scenario B), a future with mortality shocks, we employ the Mean Square Error (MSE). The MSE is an empirical measure of $\mathrm{Var}_{\boldsymbol{\mu}}[\mathrm{E}(X|\boldsymbol{\mu})]$, the forecast variance component of the decomposition, based on actual forecast errors. We use this empirical measure to analyse the share of forecast variance under a scenario that allows for the possibility of future extreme mortality or trend changes in the mortality. The MSE is based on the empirical forecast error that compares the observed cohort life expectancy $e_{0_{c,r}}^{\text{observed}}$ as published by the HMD (cohorts born from 1901 through 1932) with the mean Lee-Carter forecasts of cohort life expectancy $\overline{e_{0_{c,r}}}$. For each region r, we pool the error across all cohorts over the simulations s, resulting in the Mean Square Error:

$$\mathrm{MSE}_r = \frac{1}{33-1}\sum_c (e_{0_c}^{\text{observed}} - \overline{e_{0_c}})^2. \tag{9}$$

Classification by ages The methodological description above refers to the variance decomposition for cohort life expectancy at birth. In addition, we perform the variance decomposition for remaining cohort life expectancy at ages 40 and 80. We do so because the effect of changes in the death rates on the within-lifetable variance varies with age: it is positive for early ages, i.e. reducing infant mortality decreases lifespan inequality, but is negative for later ages and zero for the highest ages (Van Raalte and Caswell 2013). For the specification of Formulas 7, 8 and 9 for different ages of remaining life expectancy, see the additional material. Further, we put emphasis on the results for remaining life expectancy at age 40 because it is invariant to changes in early life mortality and concerns an age at which information about the future lifetime is relevant for the individual when making e.g. economic decisions.

3 Results

3.1 Variance decomposition for remaining life expectancy at age 40

How much of the total individual lifespan variance is due to the variance in the mortality forecasts and how much is due to the variance in the lifetable lifespan dispersion? The decomposition of the total variance allows us to answer this question. Under the scenario of a linear future without mortality shocks (scenario A), with the exception of Iceland, the share of the forecast variance at age 40 is very low (see Figure 1): The highest values of approximately 2.5% are observed for cohorts of the 1860s in Sweden, Norway, France, and Denmark.

We can observe reductions in the share of forecast variance to overall lifespan variance over cohorts. For all countries, the lowest share is observed for the most recent cohorts with less than 0.2%, Iceland excluded. The decline over cohorts can be explained by the increasing stability of the mortality rates that allows for better forecasts. As a result, the contribution of forecast variance to the total lifespan variance decreases.

Iceland is the smallest country in our selection with roughly 390,000 inhabitants in 2025, compared to several million inhabitants in the other countries. In smaller populations, mortality trends are more volatile and thus harder to forecast. This is reflected in the much higher share of forecast variance for Iceland. Starting around a contribution of 15% for the cohorts of the 1860s, the trend decreases drastically after the cohort 1979 until the latest cohort of 1980 for which the share of forecast variance is 5.5%.

Figure 1 also includes the country-specific Mean Square Error (MSE) which is an empirical measure of the forecast variance and thus, takes the possibility for extreme mortality events into account (scenario B). The comparison of the average MSE across cohorts with the share of the forecast variance to the total lifespan variance from scenario A without shocks reveals different results depending on country. For Denmark, England and Wales, France, and Sweden, the MSE is very similar to the share of forecast variance. For Norway (MSE = 7%) and the Netherlands (MSE = 6%), the empirical measure is slightly higher than the share of forecast variance under the scenario with no mortality shocks. Regarding Iceland, the empirical forecast error is similar to the results for the earliest cohorts. With decreasing

share of forecast variance for later cohorts, the gap between the two measures increases.

Figure 1: Share of forecast variance on total cohort lifespan variance at age 40 under a scenario without future mortality shocks (dark green) and under a scenario with mortality shocks (light green) among male birth cohorts of seven European countries.

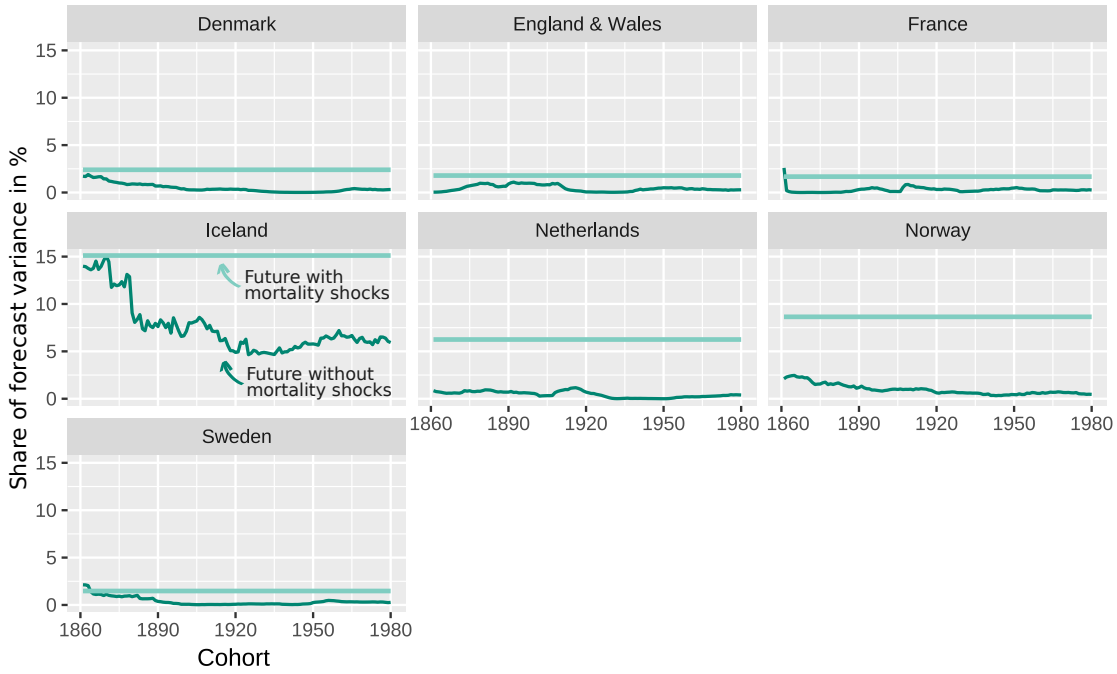
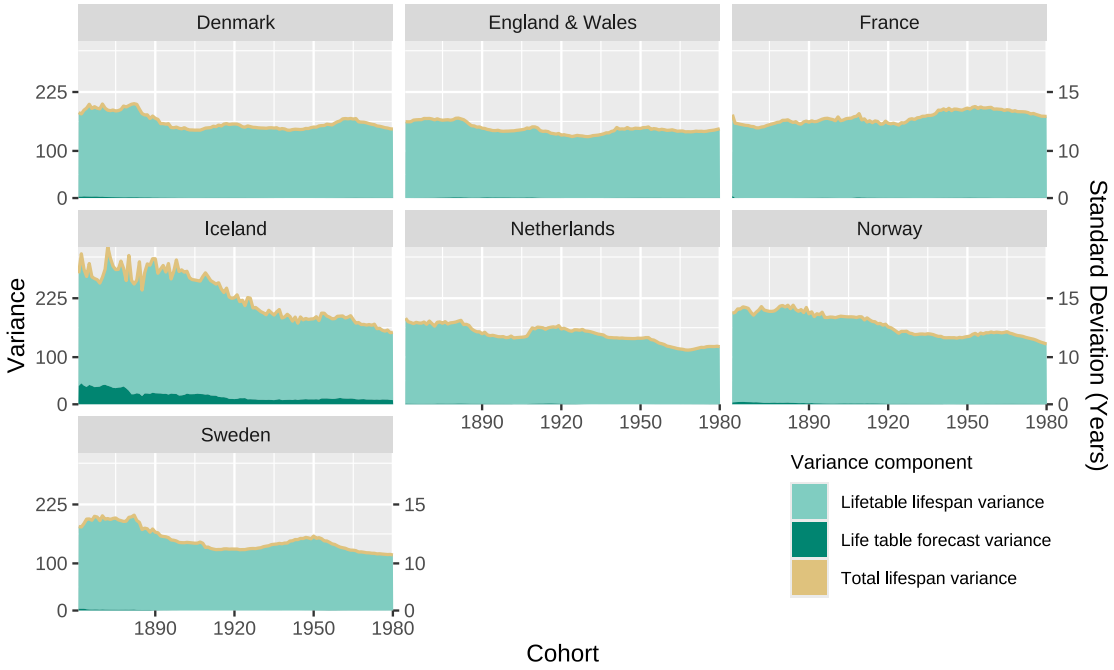


Figure 2 shows the total cohort lifespan variance at age 40 across cohorts (variance on left axis, standard deviation on right axis) split up into the two variance components from the variance decomposition under the linear mortality scenario. The changes in the total lifespan variance over cohorts are clearly driven by a reduction in the lifetable lifespan variance. For most of the countries, there is a slight decline in the total lifespan variance starting with the cohorts of the late 1800s, reaching a level below 12 years of standard deviation for cohort 1980. For the cohorts before 1900, the total lifespan variance is fluctuating between a standard deviation of 12 to 15 years, and 12 to 22 years for Iceland. With the exception of Iceland, for which declines in the forecast variance were responsible for stronger reductions in the total lifespan variance, the contribution of forecast variance to the change in total lifespan variance is small.

3.2 Variance decomposition for life expectancy at birth

We also performed the variance decomposition for male lifespan variance at birth (see Figures 3 and 4). Compared to the results for age 40, male lifespan variance at birth has a similar low share of forecast variance to total lifespan variance. The empirical error measure is slightly higher than the forecast variance component under scenario A without mortality shocks. Iceland has a more volatile pattern of the forecast component at a higher level, compared to the other countries, similar to the findings for age 40. Further, the empirical forecast uncertainty for France is much higher at 30%.

Figure 2: Variance components of total cohort lifespan variance at age 40 among male birth cohorts in seven European countries.



The total lifespan variance at age zero (see Figure 4) of all countries is on a higher level and declines across cohorts starting with the earliest cohorts of the nineteen hundreds. This can be attributed to consistent improvements in early life mortality. The change in total lifespan variance is mainly driven by reductions in the lifetable lifespan variance.

3.3 Age profile of variance decomposition

Figure 5 shows the share of forecast variance for three different cohorts for life expectancy at birth, age 40 and age 80 for a future without mortality shocks. In general, the importance of the forecast variance becomes even less important the shorter the remaining lifespan is. As a cohort ages, less of their lifetable needs to be completed to forecast the remaining life expectancy. We have seen that the total cohort lifespan variance is at a lower level for remaining life expectancy at age 40 compared to life expectancy at birth. In addition, the share of forecast variance on the total variance decreases with increasing age for which the remaining life expectancy is forecast.

4 Discussion

Under the expectation of a future with no extreme volatility in mortality, due to e.g. wars or pandemics, the results of the variance decomposition show that the overall contribution of forecast variance, or between-lifetable variance, to the total lifespan variance is low. Thus, if one does not expect crises that cause mortality shocks, the forecast variance is negligible

Figure 3: Share of forecast variance on total cohort lifespan variance at birth under a scenarios without future mortality shocks (dark green) and under a scenario with mortality shocks (light green) among male birth cohorts of seven European countries.

Figure 4: Variance components of total cohort lifespan variance at birth among male birth cohorts in seven European countries.

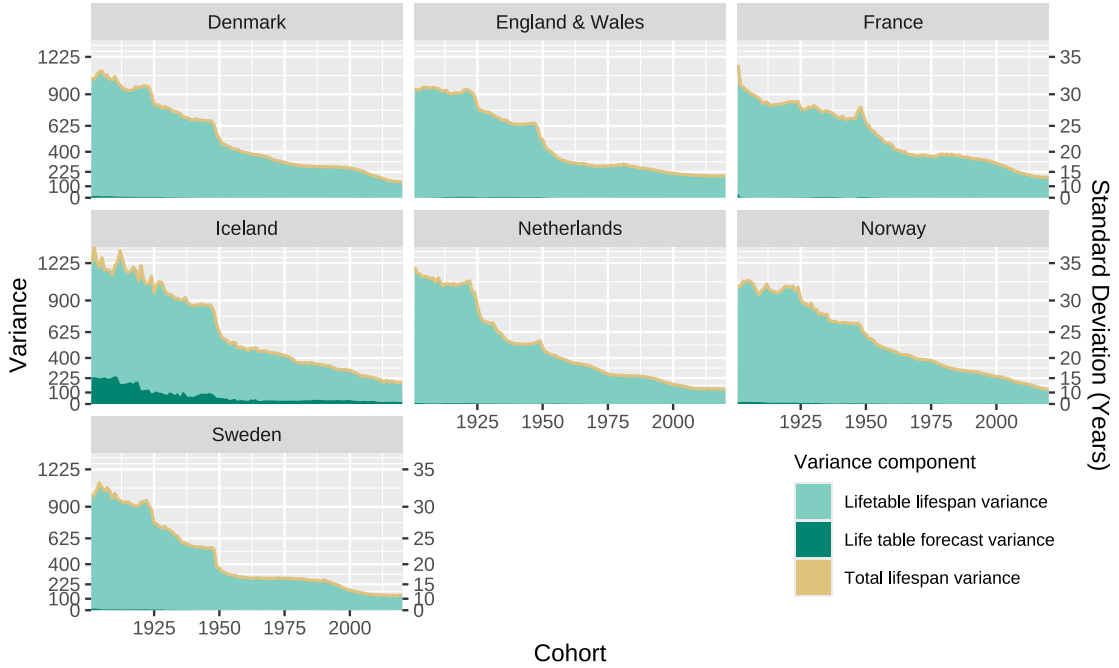
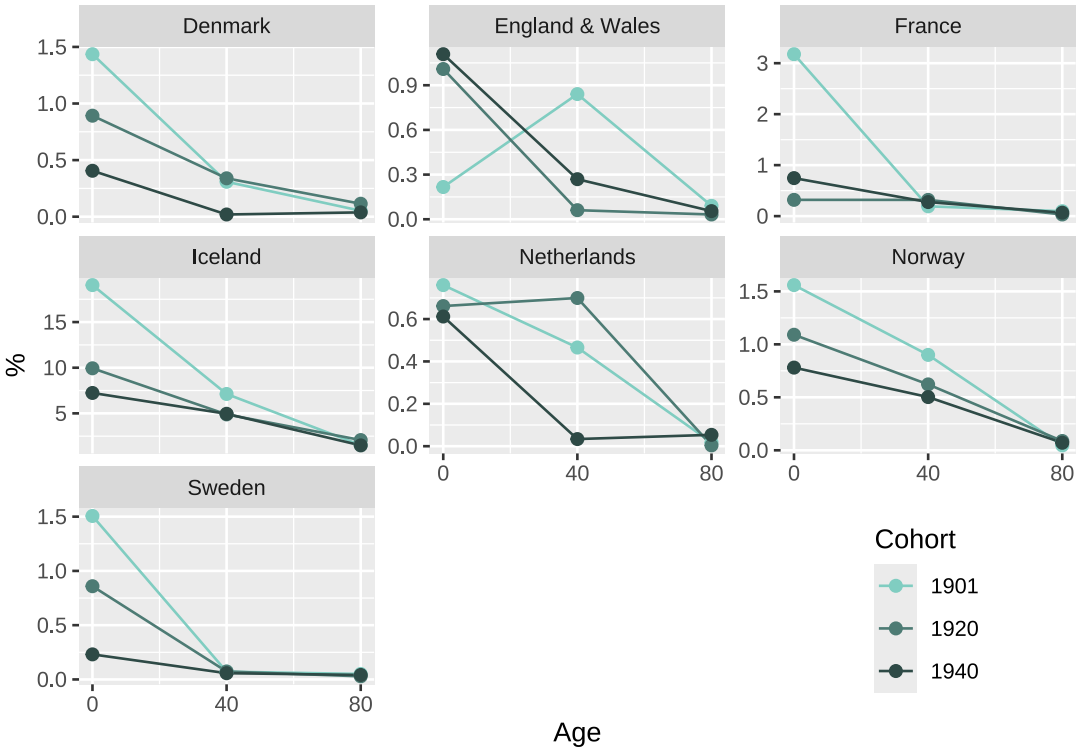


Figure 5: Share of total cohort lifespan variance over age explained by variance across forecast lifetables among selected male birth cohorts.



for the overall lifespan variance. When the possibility for extreme mortality is included, as shown by the empirical measure of forecast variance, uncertain future mortality plays a larger, but in most cases, still minor role.

Lifespan variability differs by age (Singh and Kim 2021). While lifespan variability at birth has increased due to declines in infant mortality, lifespan variability at older ages has increased because of improvements in adult survival (Engelman et al. 2014). Our results show that with increasing age of a cohort, the forecast variance contributes less to total cohort lifespan variance, due to the shorter forecasts needed to complete the cohort lifetables.

The small contribution of forecast uncertainty to lifespan uncertainty may come as a surprise. Yet, it is consistent with the approximate relative magnitudes of both quantities as regularly reported in lifespan variance calculations and forecast uncertainties. Caswell (2023) reports a lifetable lifespan variance for Swedish females in 2007 of around 100, that is, a standard deviation of 10 years. Accordingly, forecast variance around life expectancy at birth would need to be 100 as well in order to contribute half to the total lifespan variance. This is equal to a standard deviation of forecast error of 10 years which results in 95% Normal prediction intervals with a width of 2*1.96*10=39.2 years. The World Population Prospects (United Nations, Department of Economic and Social Affairs, Population Division 2024) state a 95% prediction interval of 100.98-86.30=14.68 years for period life expectancy at birth of Swedish Females in 2100. While the cohort case differs from the period case, a forecast variance high enough to match the importance of life table variance in lifespan uncertainty would be much wider than commonly reported in the demographic literature.

The life expectancy trajectories of the seven countries analyzed are characterized by sustained mortality improvements, resulting in increasing life expectancy and declining life span disparity that have changed in lockstep over time. However, particularly countries in Central and Eastern Europe have exhibited unstable mortality patterns since the 1960s. In these countries, mortality was marked by patterns that let life expectancy and lifespan disparity change independently from each other (Aburto and van Raalte 2018). This was mainly due to divergent trends of mortality change in different age groups, e.g., mortality improvements at early and old ages alongside rising mortality at mid-ages. This raises the question of whether our results are transferable to such a mortality context. Unstable mortality trajectories with, e.g., stalling life expectancy and diverging mortality development for different age groups would overall increase the total lifespan uncertainty an individual has about their age at death. Both the lifetable lifespan variance and the forecast variance would be increased, compared to a mortality regime with stable improvements. Especially when a previously stable mortality period is used as the basis for the forecasts, the forecast variance would be higher. However, the mortality experience of the cohorts 1901 through 1932 which informs our forecast error quantification was also marked by non-linearities and disruptions. Still, we have not identified a case where the forecast variance dominates the total lifespan variance even after accounting for the possibility of wars and epidemics.

There are sources of variance that we could not account for in the decomposition of the total cohort lifespan variance. First, we assume complete homogeneity in the population, that is, everyone is exposed to the same mortality rates. However, lifespan inequality differs between several socioeconomic strata, such as education, ethnicity, and region (Permanyer et al. 2018; Sasson 2016; Brown et al. 2012). Further, the true uncertainty an individual has about their lifespan is also influenced by information that is highly heterogeneous and not accessible to us. For example, individual medical histories and lifestyle factors play a role in the overall uncertainty (or certainty) about one's age at death. However, Badolato et al. (2023) find that the amount of accuracy gained by including extensive information on the individual's life when predicting individual lifespan is not substantial. Caswell (2023) shows that even with extreme levels of heterogeneity in the population, only a few percent of variance in the length of life can be attributed to inequality between groups. Instead, it is the within-lifetable variance that explains the differences in the outcomes, which is in line with our findings from the variance decomposition. Similarly, Edwards (2013) comes to the conclusion that the forecast uncertainty from Lee-Carter forecasts is small compared to the lifetable uncertainty when looking at the standard deviation in length of life above age 10 for cohorts.

Another source of variance is the likelihood of extreme events, such as wars and pandemics. The likelihood of such events plays a role in the forecast variance component of the variance decomposition. While we take these outliers into account via the empirical forecast error, future research could explore the methodology of conformal prediction (Duerst and Schöley 2024; Angelopoulos et al. 2024; Fontana et al. 2023; Shafer and Vovk 2008) to assess this type of forecast variance using the accuracy of historical forecasts. The extreme value theory is another methodological branch worth investigating in this matter (Coles et al. 2001, for an introduction). Medford (2017) argues that their research on forecasting best practice life expectancy using extreme value theory could provide valuable insights on the likelihood of life expectancy crises. Similarly, a stochastic model of extreme value mortality may be

used to obtain more realistic simulations of the forecast time-varying mortality trend term k_t of the Lee-Carter model. Wang et al. (2011) recommend the use of heavy-tailed distributions for the residuals of the Lee-Carter model and the first difference of mortality indices. This approach accounts for the possibility of extreme outliers as witnessed during, for example, wars.

5 Conclusion

Forecast variance only contributes a minor share to overall lifespan variance, under the assumption that mortality develops without extreme and prolonged period shocks. Likewise, the decline in total lifespan variance observed over more than 100 years is mostly driven by a change in the lifetable lifespan variance, and not by the drop in forecast variance. When allowing for future mortality shocks, the forecast variance still has a low share to the overall lifespan variance, with the exception of Iceland and France. Thus, these findings generally support the interpretation of lifetable lifespan variability as an indicator of lifespan uncertainty. To strengthen the case that the lifetable variance is a good proxy for individual lifespan uncertainty, other components contributing to an individual's knowledge about their eventual lifespan should be analyzed.

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Availability of data and materials

The research presented in this article is fully reproducible. We have made our codebase available to the scientific community. You can access the complete codebase, including scripts and documentation, on the website of Demographic Research alongside the publication. In addition, the data used for our analyses is sourced from openly accessible and publicly available datasets. Researchers interested in reproducing our work or conducting further investigations can freely download the data from the following link: https://www.mortality.org. The availability of open-access data ensures transparency and promotes collaboration within the scientific community.

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Additional Material for: The contribution of forecast uncertainty to lifespan uncertainty

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Conversion of period-age mortality forecasts into cohort-age forecasts

Figure 1 is a visual representation of the conversion of the period-age forecasts into cohort-age forecasts. On the example of the jump-off year 1901, we extract the forecast mortality rates from the cohort-diagonals for those cohorts being born, for those turning 40, and for those turning 80 in 1901.

Cohort variance decomposition: Specification for different ages

We perform the variance decomposition for remaining cohort life expectancy at ages $x = \{0, 40, 80\}$. We denote by X the random age at death of a cohort member and with $\boldsymbol{\mu}$ the random vector of age-specific cohort death rates parameterizing the distribution of X. We employ the stochastic cohort lifetable forecasts $\boldsymbol{\mu}_s$ to decompose the total individual uncertainty about the remaining lifespan. For brevity, we write $e_{x_s} = \mathrm{E}(X - x|X > x, \boldsymbol{\mu}_s)$ for the remaining life expectancy and $\sigma^2_{x_s} = \mathrm{Var}(X - x|X > x, \boldsymbol{\mu}_s)$ for the remaining lifespan variance at age x of a cohort member under mortality rates $\boldsymbol{\mu}_s$. For scenario A), a future without mortality shocks, we calculate the between-lifetable variance, that is the forecast variance of the remaining cohort life expectancies, as

$$\mathrm{Var}_{\boldsymbol{\mu}}[\mathrm{E}(X - x|X > x, \boldsymbol{\mu})] = \frac{1}{500 - 1} \sum_s (e_{x_s} - \overline{e_x})^2, \tag{1}$$

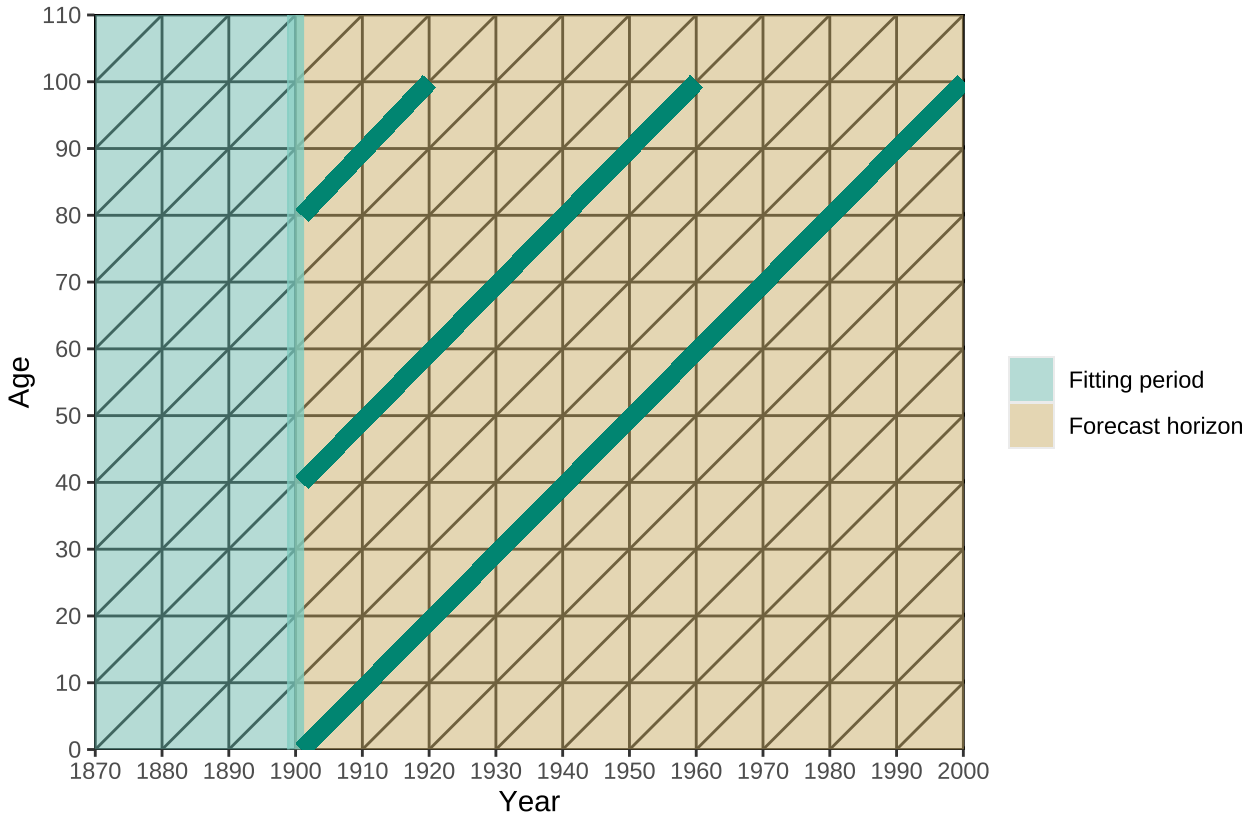


Figure 1: Visual representation of the extraction of cohort-diagonals from period mortality forecasts for the jump-off year 1901.

where $\overline{e_x} = \frac{1}{500}\sum_s e_{x_s}$ is the mean remaining cohort life expectancy forecast as averaged over the simulation runs.

The second part of the variance decomposition, the expected cohort lifespan variance, or within-lifetable variance, is calculated as the mean of the simulated lifespan variances:

$$\mathrm{E}_{\boldsymbol{\mu}}[\mathrm{Var}(X - x|X > x, \boldsymbol{\mu})] = \frac{1}{500}\sum_s \sigma^2_{x_s}. \tag{2}$$

For scenario B), a future with mortality shocks, we employ the Mean Square Error (MSE). The MSE is based on the empirical forecast error that compares the observed remaining cohort life expectancy $e^{\text{observed}}_{x_{c,r}}$ at ages x as published by the HMD (cohorts born from 1901 through 1932) with the mean Lee-Carter forecasts of remaining cohort life expectancy $\overline{e_{x_{c,r}}}$. For each region r we pool the error across all cohorts over the simulations s, resulting in the Mean Square Error:

$$\mathrm{MSE}_r = \frac{1}{33-1}\sum_c (e^{\text{observed}}_{x_c} - \overline{e_{x_c}})^2. \tag{3}$$

Influence of parameter variance on forecast variance

The contribution of parameter uncertainty to the share of forecast variance on total lifespan variance is modest. While the inclusion of parameter uncertainty in all cases increases the forecast variance, the increase does not change the interpretation of the results. Parameter uncertainty is most relevant for the small population of Iceland, where its inclusion changes the forecast contribution to lifespan variance from around 6 to 10% by 2020. We also assess the sensitivity of our results to the inclusion of parameter uncertainty for the non-penalized Lee-Carter and find that without penalties the covariance matrix has a higher chance of being ill-conditioned, making sampling impossible for some cohorts. Where parameter sampling was feasible, the impact on our results was small as well.

See Figure 3 for an example illustrating the estimated parameter uncertainty. In the penalized Lee-Carter model the variance of the estimates is concentrated in the k_t parameters. The uncertainty about the k_t (and to a lesser degree a_x and b_x) then propagates through to the drift and standard deviation of the k_t random walk.

Importance of model specification

To test the sensitivity of our results to alternative forecast model specifications we fit the Lee-Carter model without penalties as well as the Renshaw-Haberman (Haberman and Renshaw 2009) model which extends the Lee-Carter model by including an additional cohort term. Cohort life expectancy forecasts were substantially more erratic and generally higher for the Renshaw-Haberman model compared to the (Penalized)-Lee-Carter specification. The failure of the Renshaw-Haberman model to converge at all for Iceland indicates the increased demands for data to identify the added cohort effects, a demand that was not met by our 30 year fitting period. The penalized and non-penalized versions of the Lee-Carter model are mostly consistent in their forecasts with the penalized specification being less prone to

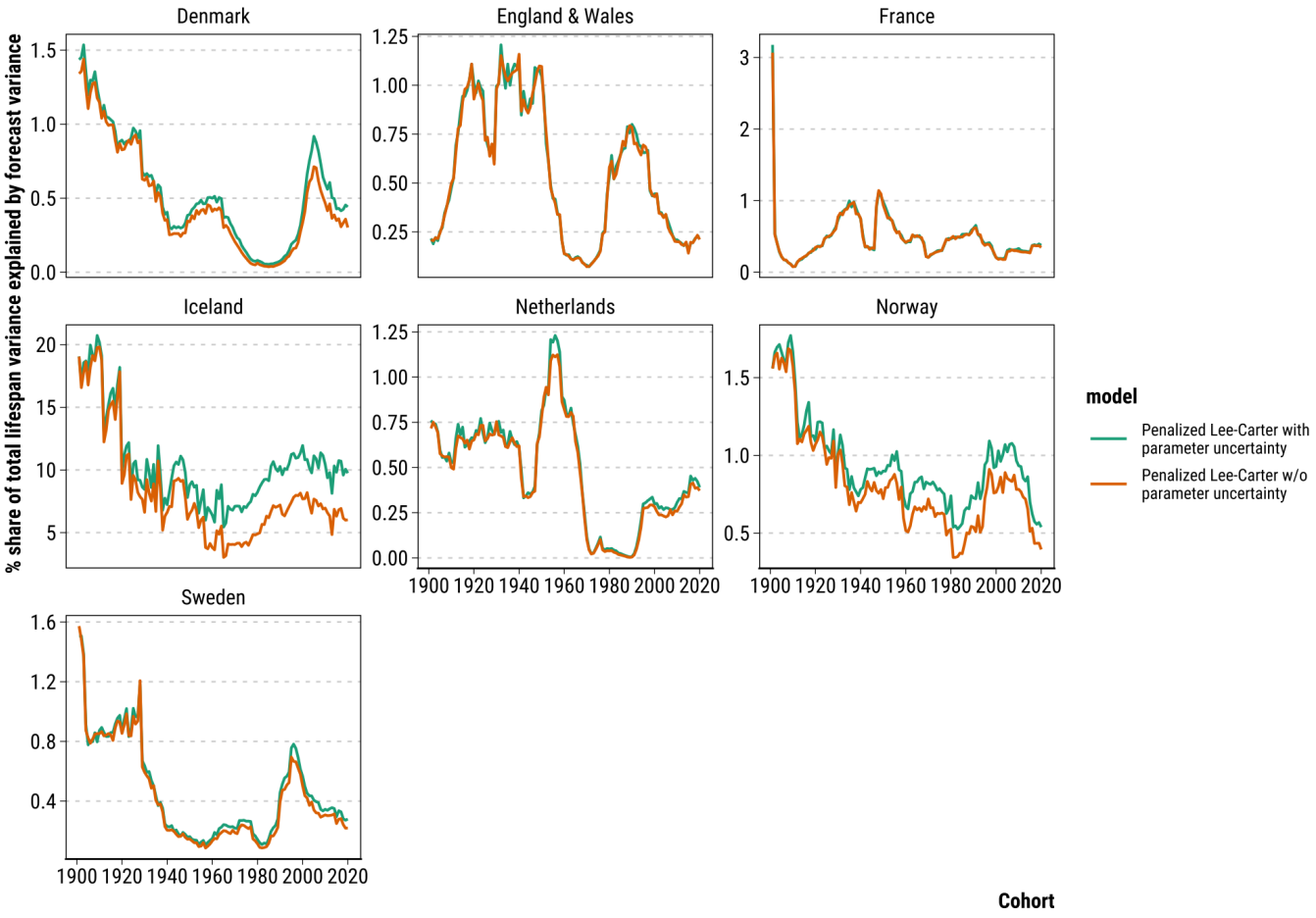


Figure 2: Forecast variance contribution to total lifespan variance with and without inclusion of forecast parameter uncertainty.

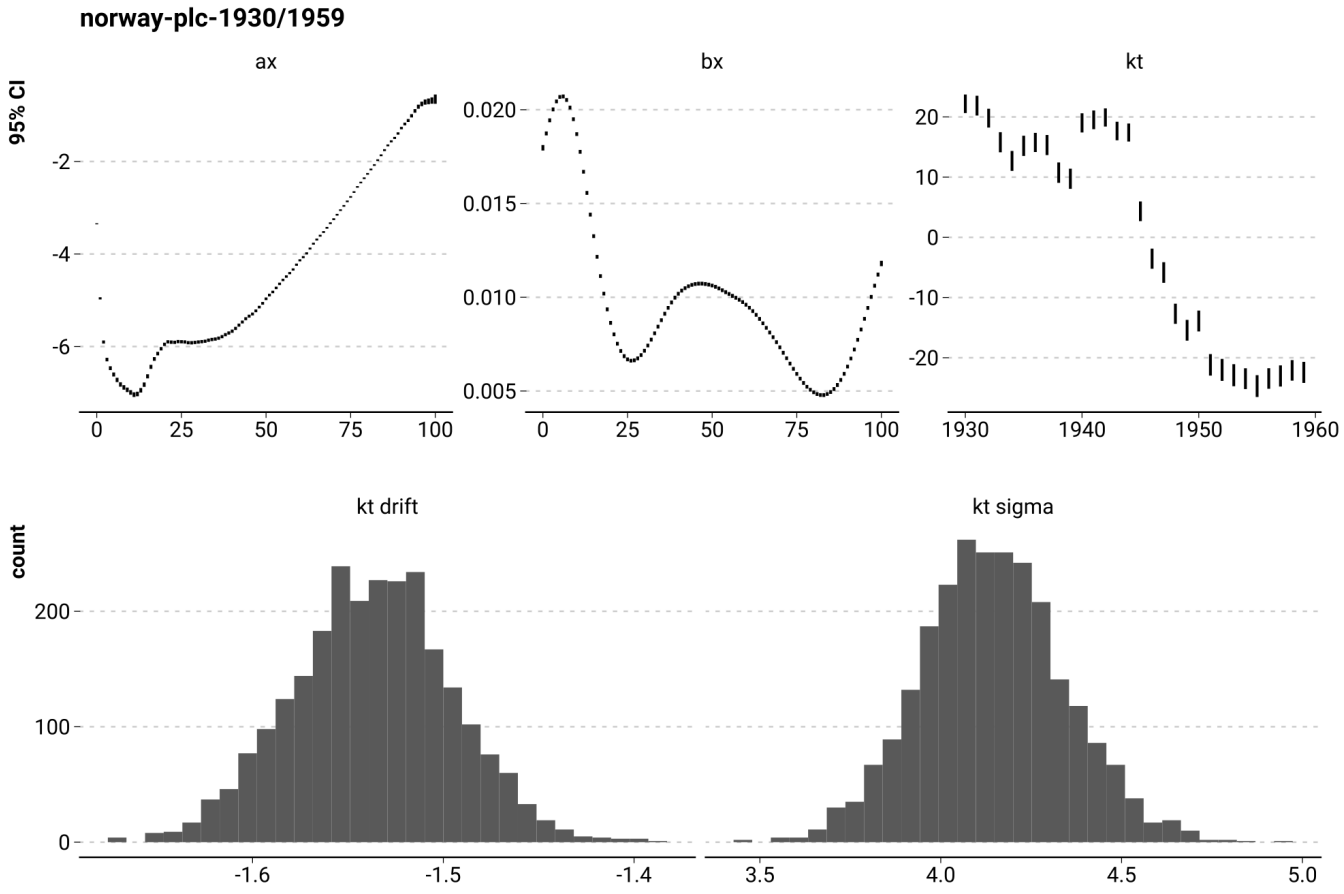


Figure 3: Distribution of penalized Lee-Carter model parameters for Norway fitting years 1930 through 1959.

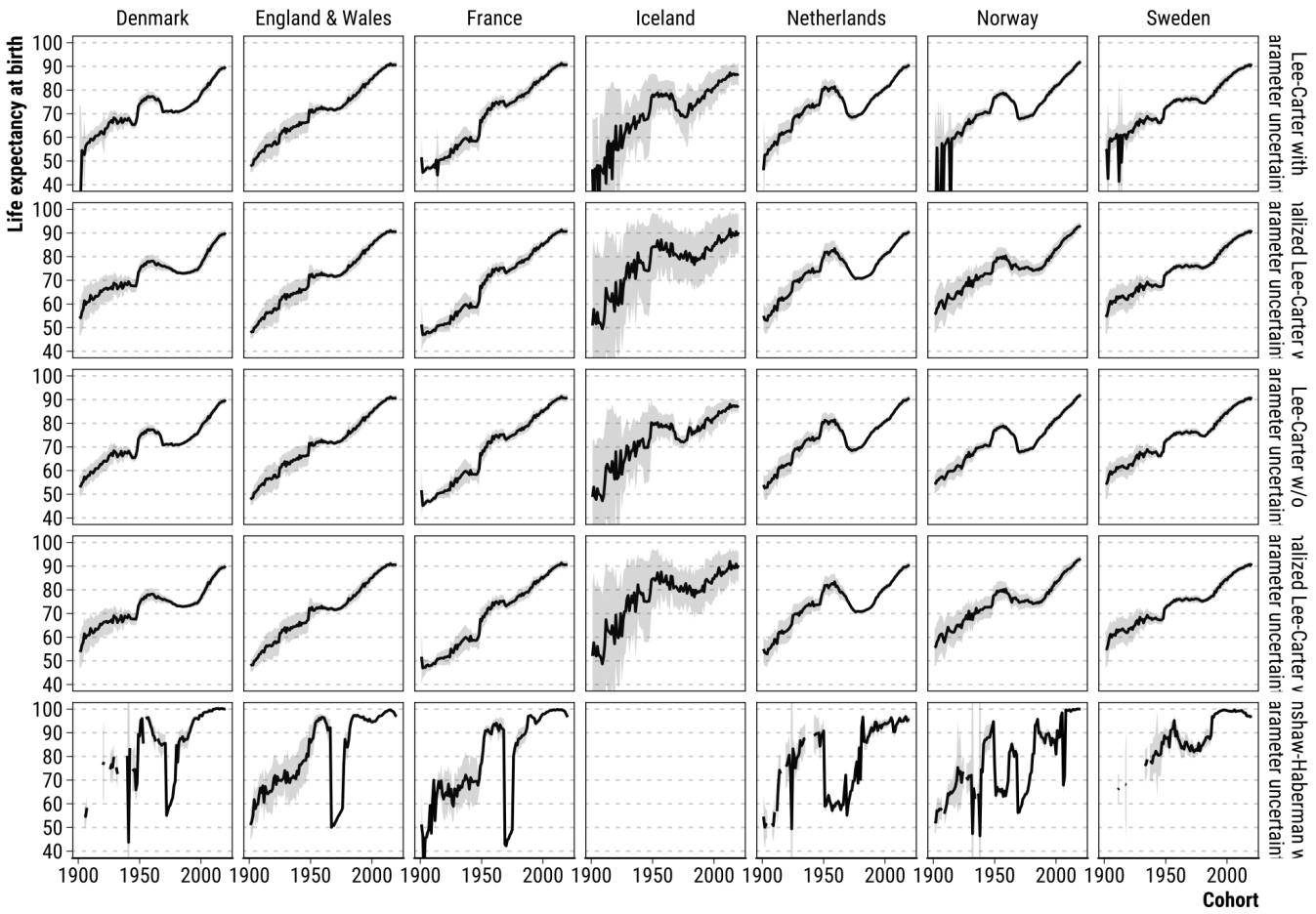
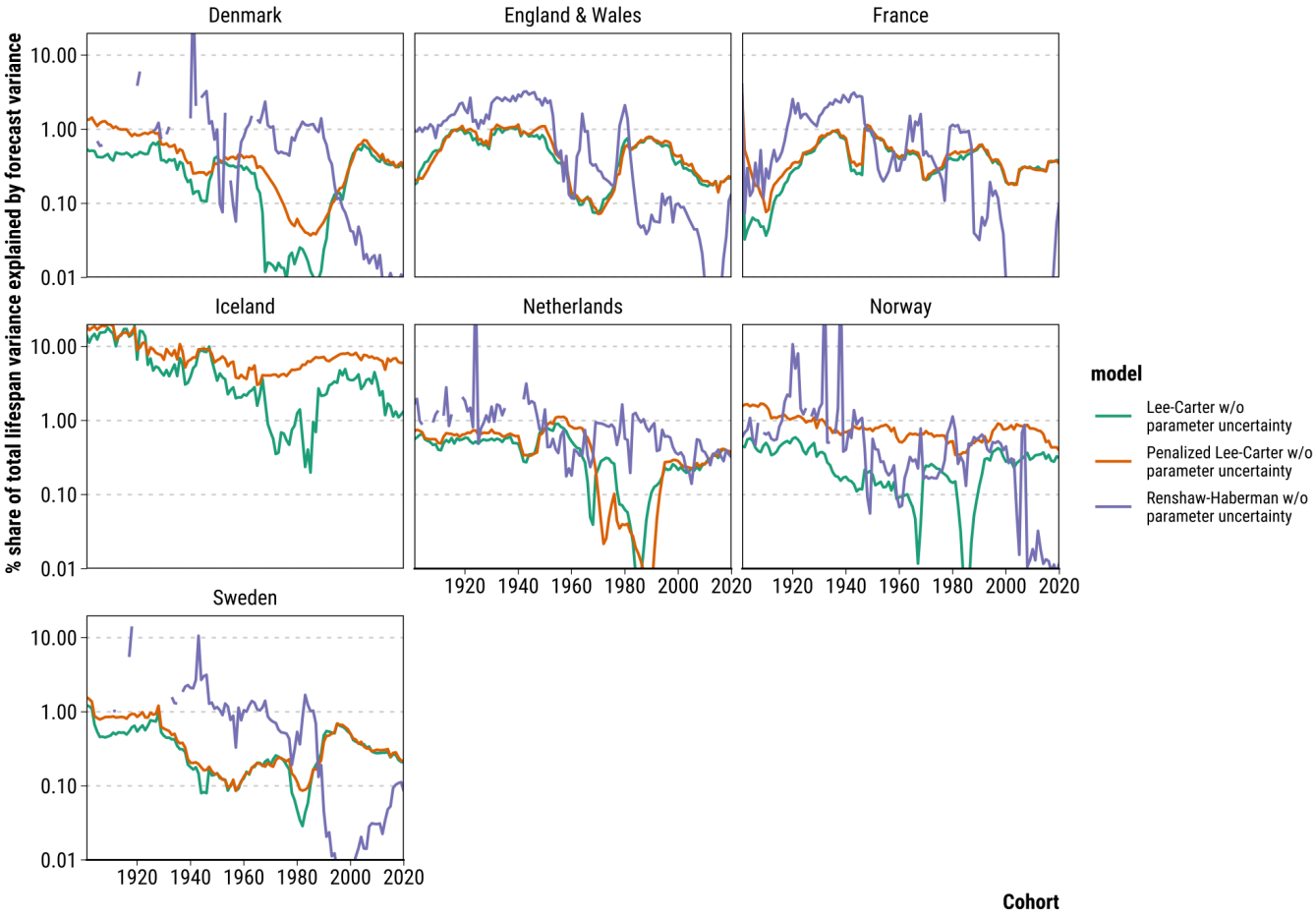


Figure 4: Cohort life expectancy forecasts 1901–2020 by model type. Grey areas indicate 95% prediction intervals.

erratic changes. As expected, the prediction intervals are slightly wider when parameter uncertainty is reflected in the forecast.

The main outcome of our analysis, the contribution of forecast variance to total lifespan variance, is largely consistent between the non-penalized and the penalized Lee-Carter specifications with the penalized version being less prone to sudden changes in trend over cohorts (see Figure 5). The Renshaw-Haberman model deviates from the Lee-Carter derived estimates with respect to the pattern over cohorts, but not in the overall magnitude of the forecast variance contribution – even for the badly specified Renshaw-Haberman model the forecast contribution to lifespan variance remains low, around 1-2%, with the exception of Iceland.

Figure 5: Forecast variance contribution to total lifespan variance by model type.



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