When All Measures Fail the Same Way: Correcting Compositional Dependence in Segregation Indices

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Abstract

The comparability of segregation measures across time and space often relies on compositional invariance—the assumption that index values remain unchanged when only group proportions vary. Despite its central role in segregation research, this assumption has never been empirically verified. Using agent-based simulations and U.S. Census data, we show that all widely used evenness indices—including dissimilarity, Gini, Theil's entropy, Hutchens' R, and Fossett's separation—exhibit a strong and nearly identical compositional dependence. We then introduce an information-theoretic correction based on information projection that isolates what segregation values would be if only composition changed, while holding neighborhood sorting dynamics constant, using *only* single-city data. This procedure provides the first empirically validated operationalization of compositional invariance, is computationally tractable, and aligns segregation indices with their intended interpretations. Applied to U.S. data, the correction reveals that the widely noted stability in White/Hispanic and White/Asian segregation since 1990 is an artifact of compositional dependence. More broadly, this work demonstrates how information-theoretic methods can formalize long-standing demographic assumptions in an empirically verifiable manner, offering a general template for methodological correction beyond segregation research.

Residential segregation by race/ethnicity is one of the defining features of American cities, with wide-reaching implications for race relations, disparities in material resources, health and well-being, access to public goods, and intergenerational mobility. Decades of demographic research have documented the causes and consequences of segregation, and developed descriptive tools for tracking patterns and trends in racial segregation across the U.S. The consensus in this scholarship is that (1) segregation between Blacks and Whites has steadily declined since 1970; (2) the segregation of Asians and Hispanics from other groups has held steady over the same period; and (3) that cities with the largest non-White populations are more highly segregated than others. Underlying these observations, however, is the belief that the measures of segregation—such as dissimilarity and separation—are insensitive to the overall composition of underlying populations, a property known as *compositional invariance*. The necessity of this invariance is especially salient given the enormous shifts in the racial/ethnic demographics of the U.S., with White population shares declining from nearly 90% in 1970 to just over 60% in 2020.

The interpretation that standard measures are compositionally invariant is often attributed to Duncan and Duncan's 1955 seminal work [1]. However, they were actually rather careful, stating that it is unclear what constitutes "distortion" when it comes to composition, and demonstrated that the dissimilarity index has an invariance in only a specific sense. James and Taeuber formulated this invariance more precisely, namely that if the population of a group becomes a factor α times its prior population in a city, the same factor α applied to all neighborhood-level populations for this group leaves dissimilarity unchanged [2]. Many other indices, such as Gini and Hutchens' R [3], were also shown to satisfy this formulation.

Further work, however, was not generally as cautious. Lieberson [4] stated that dissimilarity "avoids the influence of population composition", which represents the *intention* of compositional invariance but *not* the property demonstrated by James and Taueber. This *intention* was then echoed widely, including in Massey and Denton's classical "dimensions of segregation" work [5], Urban Institute's white paper on equity measurement [6], the U.S. Census Bureau's interpretation of segregation measures [7], and others [8, 9, 10]. Pedagogical texts have stated the *intention* of compositional invariance even more clearly, such as Howard University's introduction to the dissimilarity index stating "the value of this index is statistically independent of the relative size of the groups used in its computation" [11]. Sociological theories based on these interpretations have also been developed, with some labeling the stability of White/Hispanic segregation in recent decades as evidence of "no breakthrough in sight" [12], "especially troubling" [13], or *not* especially troubling—because there is good reason to expect that growing minority populations would result in greater discrimination [14].

Researchers' frequent (and often implicit) reliance on compositional invariance to compare segregation over time and between cities overlooks serious concerns raised by methodologists. These concerns encompass debates about what segregation measures should capture and recognition of different 'dimensions of segregation'—with only the most common dimension, known as *evenness*, having compositional invariance as a central tenet. Even when compositional invariance is theoretically desirable, methodologists have taken issue with the formulation presented by James and Taeuber as being unjustifiably arbitrary [15, 16].

Various attempts to improve upon the James and Taeuber formulation have taken the field in different directions. In 2002, Gorard and Taylor provided an alternate formulation they referred to as 'strong compositional invariance' [17]. Their proposed index to satisfy this property—known as Gorard's segregation index—has issues, such as being asymmetric and not being bounded between 0 and 1 [18]. Separately, there have been decomposition approaches, such as the work of Mora and Ruiz-Castillo [19] and Elbers [20], that partitions an index into various components using methods such as iterative proportional fitting. There have been physics-based approaches with promising predictive performance in simple simulations [21]. There have also been consideration of indices based on their compositional dependence under a random assortment of individuals into neighborhoods [22]—a scenario we refer to as 'non-interacting'. Compositional dependence in non-interacting scenarios has been referred to as index bias. Most indices, but not the *separation index* [23], demonstrate this bias, which is why the separation index is sometimes preferred.

These extensions present promising methodological approaches; however, similar to the James and Taeuber formulation, they fall short in demonstrating why their respective measures should be compositionally invariant in real cities. For example, no theoretical reasoning exists regarding why mathematical results obtained for non-interacting scenarios—used as justification for the separation index—generalize beyond this artificial case. There are multiple challenges that explain the limited progress beyond this point. (1) Accounting for the *unknown* real-world interactions is complicated, and there are no empirical tests to compare various proposed solutions. (2) Various indices of the evenness segregation dimension—including those *not* satisfying the James and Taeuber formulation—generally identify similar historical segregation trends and other comparative analyses. This apparent agreement across indices suggests that, despite the theoretical issues, serious concerns about compositional dependence are perhaps unwarranted.

Unfortunately, the consistency across evenness indices has resulted in a false sense of confidence. In this work, we demonstrate that all common measures of evenness—dissimilarity, Gini, Theil's entropy, Hutchens' R, Fossett's separation—have a strong, systematic, and virtually identical dependency on the composition. Thus, it is not that these measures are compositionally invariant, but rather that all these measures fail the same way. We first demonstrate this in a simulated agent-based setting, where we can directly alter *only* composition while keeping the city size, geographic extent, agent interactions, and neighborhood sorting dynamics constant. We then demonstrate how this dependency can be identified from data of a single city, finding similar compositional dependencies with both simulated *and* real-world city populations. These results demonstrate that James and Taeuber's formulation, Gorard's 'strong compositional invariance', as well as Fossett's separation index, are all insufficient for compositional invariance.

Additionally, we use novel methods to demonstrate that a compositionally invariant measure of segregation *can* be constructed, providing a solution to a problem that dates to at least Duncan and Duncan [1], who labeled the possibility of such a solution as "doubtful". Our "property" for compositional invariance is the standard interpretation—a measure should remain unchanged if *only* the overall composition is varied—and we validate our proposed implementation in both simulated and real-world settings. Further, rather than introducing our own index, we provide an easily-implementable adjustment to common index measures. This allows, for example, to determine dissimilarity index values at different compositions had *only* the composition been varied. Finally, we demonstrate how these compositionally invariant indices reshape our understanding of American segregation trends.

Results

Compositional dependence in agent-based simulations

The first step is to determine whether theoretical concerns about compositional invariance manifest as problems *in practice*. To this end, we examine a controlled simulated setting based on the Schelling model of segregation. The Schelling model represents seminal work in computational social science [24], demonstrating that a simple agent-based model with modest agent preferences can lead to widespread segregation. While the Schelling model does not capture real-world complexities, any measure claiming to be compositionally invariant must—at the very least—demonstrate compositional invariance in simple and controlled environments.

Our simulation consists of a 2D city with a random configuration of empty locations and agents, labeling some of the agents blue and the other agents red, and then equilibrating by allowing the agents to move for an extended period of time. The city is then parsed into 'neighborhoods' and the corresponding segregation index values are computed. We then let the simulation run for extended period of time and recompute the segregation index values, performing this process iteratively to quantify the stochastic fluctuation in index values at a fixed city composition. (See Methods for details of this approach.)

Figure 1 shows cities that differ *only* in their composition—varying between 60% to 80% blue in 5% increments—for the dissimilarity and separation indices. If composition had no

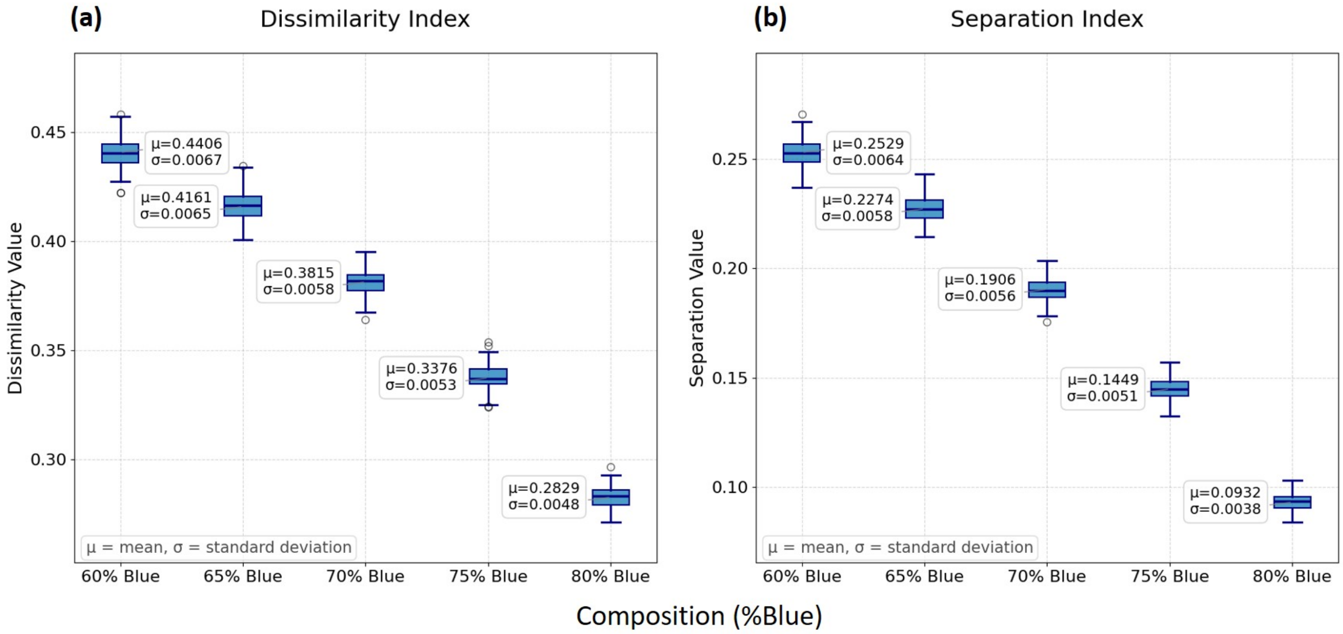

Figure 1: **Compositional dependence of segregation indices in simulated cities.** (a) Dissimilarity index and (b) Separation index values for cities with varying racial composition from Schelling model simulations. Both indices show systematic decreases as the proportion of blue agents increases from 60% to 80%, despite theoretical claims of compositional invariance. Box plots show distributions across multiple simulation runs, with mean (μ) and standard deviation (σ) values indicated. The parallel patterns demonstrate that both dissimilarity and separation exhibit similar compositional dependencies, contradicting the fundamental assumption that these indices are insensitive to the composition. (SI further shows that similar compositional dependencies are obtained for the entropy, Hutchens R, and Gini indices.)

influence on measures of segregation, then dissimilarity and separation should remain constant at various city compositions, represented by a horizontal line in Figure 1. However, not only are index values statistically different at various compositions, but the dependence is monotonic over these compositions and on the order of compositional differences. Compared to a 60% blue city, an 80% blue city has a segregation value that is on average smaller by 0.16 using dissimilarity and 0.16 using separation. Repeating this procedure with other common indices finds that an 80% blue city is also smaller on average by 0.12 using entropy, 0.09 using Hutchens R, 0.19 using Gini, and 0.12 using Gorard's segregation index[1].

We emphasize that the dependency of these evenness indices on the composition is comparable to measures *explicitly designed* to have a compositional dependence, namely those of the exposure segregation dimension. For example, the most widely used exposure measure—the isolation index, that represents the fraction of encounters a minority member has with other members of the same minority—would be 0.2 smaller for an 80% blue city than a 60% blue city in the 'random assortment' or 'non-interacting' scenario.

[1]Gorard's segregation index is asymmetric which results in additional complications. The key takeaway is that 'strong compositionally invariance' does not result in compositional invariance in testable scenarios.

Correcting for compositional dependence

Having demonstrated that all common segregation indices exhibit systematic compositional dependence, we now propose a theoretically grounded correction method based on information theory. Our approach builds directly on Barron et al. [25], who establish the core mathematical machinery and applicability across all common indices. Our approach is also similar in spirit to the iterative proportional fitting used by Elbers [20], but is substantially more tractable, yielding segregation measures that are mathematical functions of city composition.

Conceptually, our approach reformulates neighborhood-level population counts of a city into a single city-wide probability density function, and then uses information projection—a fundamental tool in information theory—to condition this distribution on a target city composition. The result of this process (known as exponential tilting in statistics or Boltzmann suppression in physics) theoretically accomplishes exactly what compositional invariance aims to achieve: namely, it isolates what segregation values would be if *only* the city's composition changed given the available data. After computing adjusted index values (index values evaluated at the same target composition) researchers can draw comparisons between places and across time that are not influenced by compositional differences. (See Methods for details of this procedure.)

To show the utility of this approach, Figure 2 presents adjusted segregation indices using our simulated cities. The or-

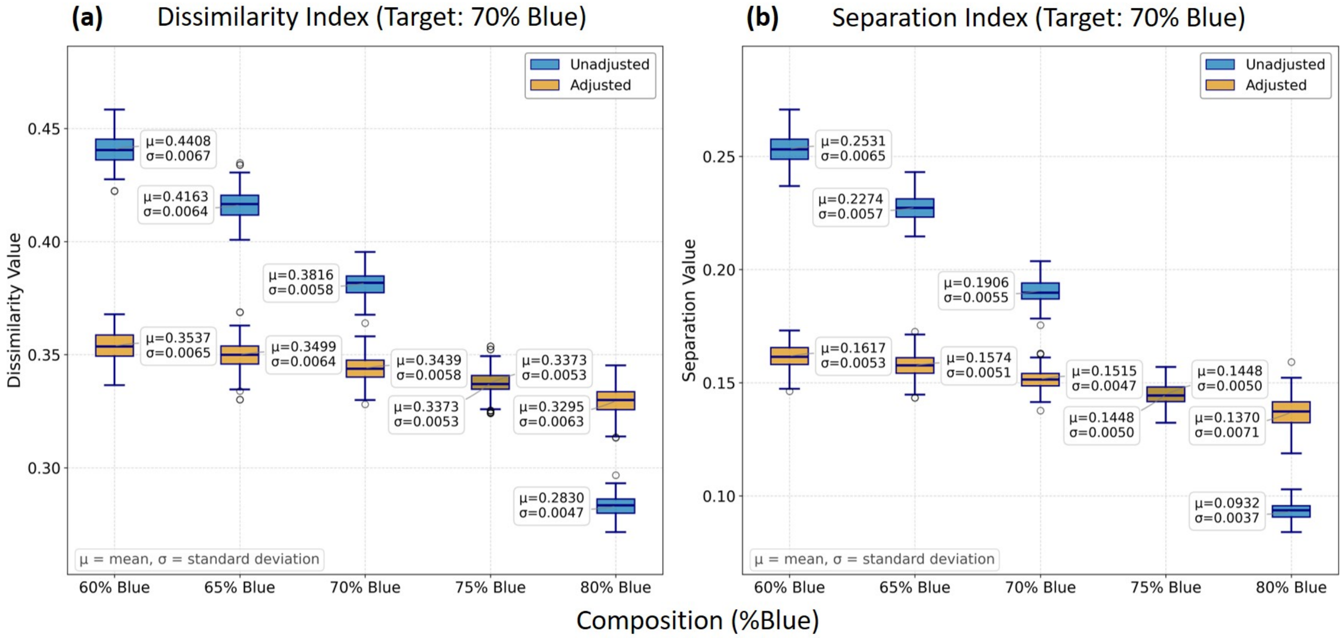

Figure 2: **Compositional adjustment substantially reduces bias in segregation indices.** (a) Dissimilarity index and (b) Separation index values before (blue) and after (orange) adjustment to a target composition of 75% blue agents. For the adjusted values, we applied our correction method to each simulated city independently, estimating what its segregation would be if it were 75% blue. Box plots show the distribution of these estimates across multiple simulation runs. By design, unadjusted and adjusted values are identical at the target composition (75% blue), confirming the mathematical consistency of the adjustment procedure. While unadjusted indices exhibit strong compositional dependence (declining by approximately 0.16 from 60% to 80% blue), the adjustment procedure substantially reduced this bias, with segregation index values identified within 0.02 of the 75% blue city values on average, regardless of the actual city compositions.

ange box plots in each panel reflect the dissimilarity and separation values that are obtained for a target composition of a 75% blue city. We emphasize *each simulated city is adjusted independently*, identifying their segregation values at a different composition (75% blue) using only single-city data. In contrast to the strong compositional dependence of around 0.16 when indices are computed directly (blue box plots), we find that cities that are 60% blue or 80% blue allow us to identify index values for a 75% blue city to within 0.02. (Similar results are obtained for the entropy, Hutchens' R, and Gini indices).

Empirical validation on real-world cities

Agent-based models help us to diagnose the effects of compositional variation on indices, keeping all other aspects constant. However, the real-world determinants of segregation are more complex and varied than what these models can capture. Individuals' race-related preferences of course matter, but also the availability of homes across neighborhoods as well as "place-stratifying" institutional processes that calcify racial-spatial boundaries.

Each city can be thought of as having its own *signature architecture of segregation*—and it is in the interest of researchers to understand how this architecture varies across cities and why it changes over time. Our results of Figure 2 demonstrate that a consistent signature can be obtained in a Schelling model despite the presence of compositional differences. However, it remains to be demonstrated that a consistent signature can be obtained in real cities, the relevant scenario in which segregation measures are applied. This demonstration is particularly important given that prior compositional invariance formulations lacked similar validation, and why in hindsight they have proven problematic.

To demonstrate compositional invariance in real cities, We first construct a partial replica of a city that draws (with replacement) a subset of neighborhoods from the overall city. Naturally, this subset is likely to reflect the city's composition and underlying segregation architecture—but its composition (*e.g.,* percent White across all neighborhoods in the subset), and computed segregation index value, will vary in each subset. After repeating this subset exercise many times, we can identify index values that arise for subset compositions near a specific values (*e.g,.* 60% White). If the segregation values are the same regardless of the subset composition, then the segregation measure is demonstrating compositional invariance. Conversely, if there is a systematic dependence of index value on subset composition, then the segregation measure is demonstrating compositional dependence.

Figure 3a demonstrates the observed compositional dependency using 100-million neighborhood subsets (blue line) for the dissimilarity index, obtained using a *single* Schelling-simulated 70% blue city. We note that this dependency is consistent with our earlier simulation results, Figure 1, where *only* the composition was varied directly. Importantly, this empirical test is by design applicable to real cities. Figure 3b demonstrates the analogous result using subsets for White/Hispanic segregation in 2020 Chicago—where a comparably strong compositional dependency is found.

Most importantly, we now demonstrate that we can reliably obtain a consistent segregation signature for subsets that are drawn from the same city, despite the presence of compositional differences. For our adjusted dissimilarity values, we set the target composition to the actual city-wide composition (70% blue in the simulated city and 68% White in Chicago). We emphasize that in all cases *each subset is adjusted independently*, and only after the adjustment are results for subsets of similar compositions aggregated to obtain a measure of the mean and variance. The result for Chicago demonstrates that we can use, for example, a 60% White subset *or* 80% White subset to reliably identify the *actual* dissimilarity values that arise at a subset composition of 68% White to within 0.01 on average. This is despite the 60% White and 80% White subsets differing in their dissimilarity by around 0.15, on average, had no compositional adjustment been performed. This suggests that if Chicago was 80% White, and then changed to being 60% over time—*without any change in attitudes or neighborhood sorting dynamics*—its dissimilarity value can be expected to go up by 0.15. We note that comparable results are obtained using separation, entropy, Hutchens' R and Gini indices, using other pairwise comparisons (White/Black or White/Asian), and similarly consistent signatures—but that are distinct from Chicago—are obtained for other cities..

Updated Historical Trends in Segregation

To examine a substantive implication of these adjustments for segregation research, we consider how our understanding of segregation trends differs once compositional dependencies are removed. The traditional (unadjusted) and adjusted trends American trends are shown in Figure 4 for the dissimilarity and separation indices, representing results aggregated over many U.S. cities and with the target composition set to each city's historical 1990-2020 average composition. (See Methods for details of this analysis.)

The implications of adjusting for compositional change are immediately clear: when the systematic dependence of segregation indices on the composition is removed, the trends in White/Hispanic and White/Asian segregation deviate substantially from the consensus trends. In fact, the decline in White/Hispanic segregation is found to outpace the decline in White/Black segregation over the last several decades. Specifically, compared to -0.05 decline that would be obtained with traditional dissimilarity for White/Hispanic segregation between 1990 and 2020, we observe a compositionally invariant decrease of -0.21. Likewise, White/Asian segregation shows a shift from an increase of 0.01 to compositionally invariant change of -0.12. In contrast, the Black population, which has remained relatively stable in its compositional share, sees a minor adjustment from a decrease of -0.11 to a decrease of -0.14.

Trends with other indices, such as separation (Figure 4b), as well as entropy, Hutchens' R, and Gini show similar results. In short, the relative stability of White/Hispanic and White/Asian segregation since 1990—a widely accepted finding of the segregation literature [12, 26]—appears to be driven entirely by the rapid growth in these populations and by the counterbalancing compositional dependence of *all* common segregation indices.

Discussion

Our findings demonstrate that widely accepted segregation comparisons, between cities and over time, are confounded by compositional differences and do not directly reflect changes attitudes, policies, or other neighborhood sorting dynamics. The effect of this confounder is large, systematic, and similar for all common evenness measures, providing a plausible explanation for why theoretical concerns did not result in caution in the broader literature. When all measures fail the same way, it creates a false sense of confidence.

The compositional dependence we identify has profound implications for interpreting American segregation trends. The consensus that White/Hispanic and White/Asian segregation remained stable between 1990 and 2020 appears to be an artifact of rapid demographic diversification and systematic compositional dependencies of common indices. When measures are aligned with their standard interpretations, we observe substantial declines in segregation for all comparisons, with Hispanic segregation declining on levels similar to, or even more rapidly, than Black segregation. This fundamentally alters our understanding of American residential integration and suggests greater progress toward integration than previously recognized, as well as a lasting exceptionalism of White/Black segregation.

Our approach represents the first empirically verified operationalization of compositional invariance that incorporates realistic interaction patterns. Unlike previous theoretical approaches relying on non-interacting scenarios, our method works within actual cities and is able to identify a consistent *signature architecture of segregation* for subsets drawn from the same city despite compositional differences. We emphasize that it was not *a priori* obvious that this could be accomplished and, admittedly, it is perhaps surprising that it can.

Rather than introducing a new index, the compositional adjustment presented in this work can be applied to all common evenness indices. These 'adjusted' indices should not be thought as an entirely new set of indices, but rather an alignment with their already-assumed interpretation of being compositionally invariant. That traditional evenness indices are compositionally invariant is not an interpretation that can be continued to be made given the results of this work.

Finally, these findings highlight a critical lesson for science more broadly: the agreement of many measures does not remove the possibility of substantial measurement error, and this possibility has the potential undermine decades of research. For segregation research specifically, the presence of large compositional dependencies requires comprehensive re-evaluation of comparative analyses and historical trends, with our analyses

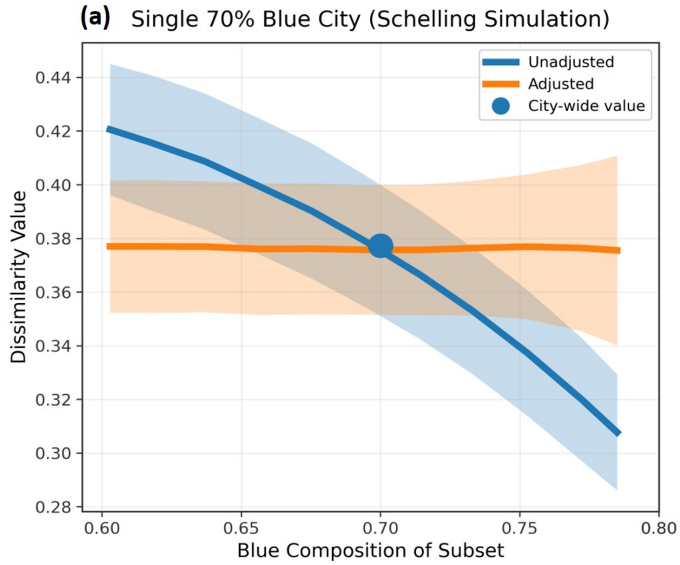

Figure 3: **Single-city empirical test reveals compositional dependence in both simulated and real cities.** (a) Single Schelling-simulated 70% blue city and (b) 2020 White/Hispanic Chicago, both analyzed using random 100-neighborhood subsets. For each subset, we independently calculate the dissimilarity index and apply our compositional adjustment method. Subsets with similar compositions are then binned together to compute mean values and standard deviations. The solid blue lines show mean unadjusted dissimilarity values, which decline systematically as subset composition increases and demonstrates a strong compositional dependence. The solid orange lines show mean adjusted dissimilarity values after each subset is independently adjusted to the city-wide composition, virtually eliminating the compositional dependence. Shaded regions indicate standard deviations of dissimilarity values within each composition bin. Blue dots mark the city-wide dissimilarity values at the overall city composition (70% blue for simulation, 68% White for Chicago). The parallel patterns represent similar compositional dependence for the dissimilarity index in both a simulated city and Chicago, and that our adjustment method effectively reduces this bias—allowing to obtain consistent measures of segregation despite compositional differences.

suggesting that American cities have achieved substantially greater residential integration than widely believed.

Methods

Data

We used decennial Census data from the IPUMS NHGIS for 1990-2020 at the block group level. We defined racial groups as non-Hispanic White, non-Hispanic Black, and Hispanic (any race), following standard segregation research conventions. Only individuals who did not indicate multiple races were included in the White, Black, and Asian groups. Anyone who indicated 'Hispanic' was treated as Hispanic. Until 2000, 'Asian' and 'Pacific Islander' were grouped together, so for consistency with 1990, 'Asian' indicates the combined grouping of 'Asian' and 'Pacific Islander'.

To avoid bias from institutional populations, we excluded block groups where more than 25% of residents lived in group quarters. Metropolitan areas (referred to as cities in the main text) were included only if they contained at least 1,000 individuals of each group (White, Black, Hispanic, Asian) in all decades.

For historical trend analysis, we applied additional filters to ensure reliable compositional adjustments. First, we required that the minority group under consideration constituted at least 3% of the total population in each decade from 1990 to 2020. Second, we included only metropolitan areas that had at least 25 block groups with compositions on each side of the target composition (each metro's four-decade average composition). This filtering resulted in 157 metropolitan areas for White/Black comparisons, 88 for White/Hispanic comparisons, and 24 for White/Asian comparisons. When aggregating results across metropolitan areas, all metros were weighted by their total population to produce national-level trends.

Agent-Based Schelling Simulations

We implemented a Schelling-like segregation model on a 1000×1000 grid with red and blue agents. At each time step, 10% of agents were randomly selected to potentially relocate. For each selected agent, we calculated happiness as the fraction of same-type agents among the 8 immediately adjacent cells (Moore adjacency, including diagonals), excluding empty locations from the fraction calculation. Agents probabilistically moved to random empty locations based on happiness improvement using a sigmoid function: $P(\text{move}) = 1/(1 + e^{-s \cdot \Delta h})$, where $s = 3.33$ is the sensitivity parameter and Δh is the happiness improvement.

To test compositional dependence, we varied racial composi-

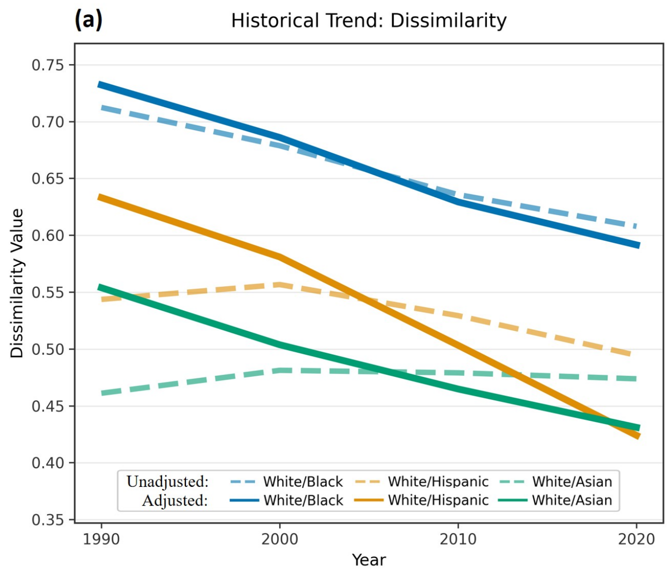

Figure 4: **Compositional adjustment reveals substantial declines in all segregation measures.** Historical trends from 1990-2020 for (a) Dissimilarity index and (b) Separation index. Dashed lines show traditional (unadjusted) trends: modest decline in White/Black segregation, with White/Hispanic and White/Asian segregation remaining relatively stable. Solid lines show compositionally-adjusted trends, with each city independently adjusted to its four-decade average composition before aggregation. After accounting for compositional dependence, all group comparisons exhibit substantial declines, with Hispanic segregation declining more rapidly than Black segregation. The apparent stability in White/Hispanic and White/Asian segregation since 1990 is revealed to be an artifact of rapid demographic growth in these populations combined with systematic compositional dependence in segregation indices. Results include cities meeting population and neighborhood distribution requirements for all decades (see Methods).

tion from 60% to 80% blue in 5% increments while maintaining constant total population density (62.5% occupied), city size, and interaction parameters. Simulations largely equilibrate in around 10, 000 steps. For each composition, we used the city state every 10, 000 steps, from step 50, 000 onward, with around 150 states at a given city composition.

For segregation index calculations, we partitioned each simulated city into neighborhoods using a consistent random spatial partition across all composition variants. Neighborhoods averaged 500 locations with a standard deviation of 150.

Single-City Empirical Test for Compositional Dependence

To empirically test compositional invariance using data from a single city, we randomly sampled 100 neighborhoods with replacement and computed segregation indices for each subset. We repeated this process 100 million times per city to obtain subsets with a wide variety of compositions for analysis. This approach allows us to examine whether segregation measures computed from random subsets depend systematically on subset composition—evidence of compositional dependence.

Information-Theoretic Correction Method

To address observed compositional dependence, we developed a correction method based on information theory. We followed Barron et al. [25] to transform neighborhood-level population counts into a probability density function (PDF), $P(\pi)$, representing the probability that a randomly selected individual resides in a neighborhood with racial composition π (*e.g.,* 60% White).

Barron et al. [25] have further demonstrated that all common measures of evenness and exposure can be computed from this distribution formulation. For example, the dissimilarity and separation indices are given by

$$D = \frac{1}{2\bar{\pi}(1-\bar{\pi})} \sum_{\pi} P(\pi)|\pi - \bar{\pi}| \tag{1}$$

$$S = \frac{1}{\bar{\pi}(1-\bar{\pi})} \sum_{\pi} P(\pi)(\pi - \bar{\pi})^2, \tag{2}$$

where $\bar{\pi}$ is the city-wide composition.

To condition this distribution on a different city-wide composition, we use information projection. Let μ represent the target composition, the result of this procedure is

$$P(\pi|\mu) \propto P(\pi)e^{v\pi},$$

where the parameter v is identified from the relation $\sum_{\pi} \pi P(\pi|\mu) = \mu$. Note that only one value for v satisfies this relation for any given μ.

What makes information projection powerful is that $P(\pi|\mu)$ is *guaranteed* to better represent *every* city (or neighborhood subset) having the composition μ than $P(\pi)$ (outside of the trivial case that $\sum_\pi \pi P(\pi) = \mu$ is already satisfied.) This improvement is guaranteed using the Kullback-Leibler (KL) divergence measure and, equivalently, likelihood approaches, and follows from the Pythagorean identity of information theory.

The adjusted indices are then obtained by replacing $P(\pi) \to P(\pi|\mu)$ and $\bar{\pi} \to \mu$, and can be written as the more-familiar sum over neighborhoods using

$$D(\mu) = \frac{1}{2\mu(1-\mu)} \frac{1}{\sum_i t_i e^{v\pi_i}} \sum_i t_i |\pi_i - \mu| e^{v\pi_i} \quad (3)$$

$$S(\mu) = \frac{1}{\mu(1-\mu)} \frac{1}{\sum_i t_i e^{v\pi_i}} \sum_i t_i (\pi_i - \mu)^2 e^{v\pi_i}, \quad (4)$$

where i indicates the neighborhood, t_i the neighborhood's pairwise population, π_i the neighborhood's pairwise composition, and $\mu = \frac{\sum_i t_i \pi_i e^{v\pi_i}}{\sum_i t_i e^{v\pi_i}}$.

Optimization can be avoided by evaluating μ, and corresponding segregation index at μ, while varying v from a large negative value to a large positive value. Given that monotonic dependence between v and μ, this will identify the segregation index-function (e.g., $D(\mu)$) as a function of μ. Identifying the segregation index value at composition of interest then identifies the adjusted index value at this target composition.

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