Implicit pension debt in pay-as-you-go pension systems: the stationary and balanced case

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Abstract

This paper revisits the concept of the implicit pension debt (*Ipd*) in pay-as-you-go (Paygo) pension systems and proposes a few innovations. The first is the representation on a Lexis diagram of all the four versions of *Ipd* debated in the specialized literature: this greatly helps non-specialized readers to better understand their meaning and the interconnections between them. This framework leads to the second and most important original contribution of this paper. Under stationary and balanced demographic and economic conditions, *Ipd* can be expressed as the product of three factors: the average pension benefit (*P*), the number of pensioners (*S*), and the average age gap (*D*) between when contributions are paid and pensions are received. The proposed formula sheds new light on several aspects of Paygo systems, including the quasi-capital gains (losses) they are known to produce during expansion (contraction) phases. A brief discussion suggests that these findings and considerations apply, albeit only approximately, to a much broader range of cases than the stationary and balanced one examined here.

Finally, the *Ipd* concept indicates that Paygo systems would be substantially more resilient to demographic changes if they included also child benefits, and not only pensions. While such a transformation would arguably lead to an improvement in the long run, its realization is unlikely due to the high transition costs. In all cases, the proposed framework supports clearer diagnostics and better-informed policy responses to the challenges that demographic ageing and decline pose to pension systems.

Keywords

Implicit pension debt, Pay-as-you-go pension systems, Stationary population, Quasi capital gains and losses

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1. Introduction and purpose

Paygo (pay-as-you-go) pension arrangements create and implicit pension debt (*Ipd*) – also referred to as the unfunded pension liability – which can be defined as "the present value of pension promises, net of future pension contributions, that are implicit in the current legislation. … The implicit pension debt quantifies the cost of closing a Paygo pension scheme while fully honouring all past pension promises" (Beltrametti and Della Valle, 2011, p. 2).

At first glance, this definition appears straightforward, yet it gives rise to at least four distinct interpretations, with their corresponding empirical measures. To date, I have found no presentation of these measures with reference to the Lexis diagram, which, in my view, greatly aids comprehension of their significance and their differences, and paves the way to the novel formula proposed in section 3. One of this paper's innovations therefore lies in providing such a graphical exposition of the four possible interpretations of *Ipd* (Section 2).

A second innovation is the application of the *Ipd* concept to the "fully stationary and balanced" case, i.e. a state where:

1. Demographic and economic variables remain constant (a stationary population,[1] unvarying wages, employment rate, prices, etc.).
2. The Paygo pension system is perpetually in equilibrium, with benefits disbursed exactly matching contributions received.

Under this theoretical scenario, three of the four *Ipd* interpretations coincide (those pertaining to the entire population), and the *Ipd* may be expressed in a remarkably simple – and, to my knowledge, novel – form, i.e. as the product of three term: number of beneficiaries, average benefits, and the distance between two average ages: at receiving benefits and at paying contributions. This formula, which constitutes the paper's main novelty, also offers fresh insights into the established concept of quasi capital gains and losses, as will be discussed (Section 3).

Next, I contend that the fully stationary and balanced case under examination here is not merely an academic abstraction. When Feldstein introduced the notion of *Ipd* in 1974, pension systems were markedly unbalanced: pension promises per pensioner outstripped contributions, thereby creating resources, which Feldstein described as "net social security wealth", apparently from thin air. This seeming "miracle" stemmed from quasi capital gains driven by two prevalent historical trends: demographic expansion and steadily rising pension coverage – i.e. an increasing proportion of the population enrolled in social security. Today, both trends have stalled or even reversed: many developed countries now face stagnant or declining populations, and calls to leave the public Paygo pension system (reducing coverage) are growing louder, as erstwhile quasi capital gains threaten to morph into quasi capital losses. Consequently, also policymakers, and not only scholars, are paying far greater attention to pension design, emphasising viability, sustainability, and long-term equilibrium. Although these objectives remain unmet, today's policy and demographic environment is nearer than ever to the stationary and balanced scenario discussed herein (Section 4).

The discourse surrounding the precise interpretation of *Ipd*, its ideal magnitude (if such exists), and the implications of an "excessively large" *Ipd* remains ongoing. Although even a substantial *Ipd*

[1] This is a theoretical population whose age structure coincides with the L_x series (years of life lived) in a life table – or with the l_x series (survivors) in the continuous case. In a stationary population, everything is constant (births, fertility, deaths, mortality, total population and population by age), while migration is absent.

may be sustainable (and indeed is sustainable in the stationary and balanced scenario outlined here), it poses significant risks, as it directly correlates with potential quasi capital losses. Consequently, compelling reasons exist to mitigate *Ipd*. In the paper's concluding section, I exploit the novel formula to show that reconfiguring pension schemes into genuine intergenerational transfer mechanisms – thereby allocating benefits also to younger cohorts – substantially curtails *Ipd* and fortifies the system against diverse future demographic perturbations (Section 5).

Regrettably, any diminution of *Ipd* corresponds on a one-to-one basis with the realisation of a quasi capital loss. Proposing immediate quasi capital losses to preclude larger future losses may appear paradoxical or futile, given that the vast majority of the electorate would resist such measures. Yet historical precedent shows that similar objections were raised against pension retrenchments presumed to be politically unviable, only for them to be enacted in practice (Kohli and Arza, 2011; OECD, 2023). The most plausible rationale for this paradox is that resisting reform of an unsustainable pension system risks precipitating its collapse: in such circumstances, accepting a modest, immediate loss may be preferable to confronting the non-negligible probability of a much larger future loss. Analogously, incurring a manageable quasi capital loss now to avert more substantial losses later may be a judicious strategy.

Ultimately, however, my objective here is to persuade readers that intergenerational transfer schemes where transfers benefit both the younger and the older generations have distinctive advantages over "traditional" pensions schemes, where resources are transferred only "forward", from the adults to the older adults. Whether such a transformation is practicable in reality remains an open question.

2. A demographic perspective on *Ipd*

For simplicity, let us focus on three primary age groups: the young (Y), up to age α (for example, 15 years); adults (A), between ages α and β (for example, up to 65 years), and older adults, or seniors (S), above age β. Let us also assume that the employed population (E) predominantly falls within the adult cohorts (with $e=E/A$ denoting the employment rate, typically around two-thirds in OECD countries), whilst seniors are synonymous with retirees receiving an average pension benefit P (see Table 1 for the list and definitions of the symbols used in this paper - some of which introduced later on).

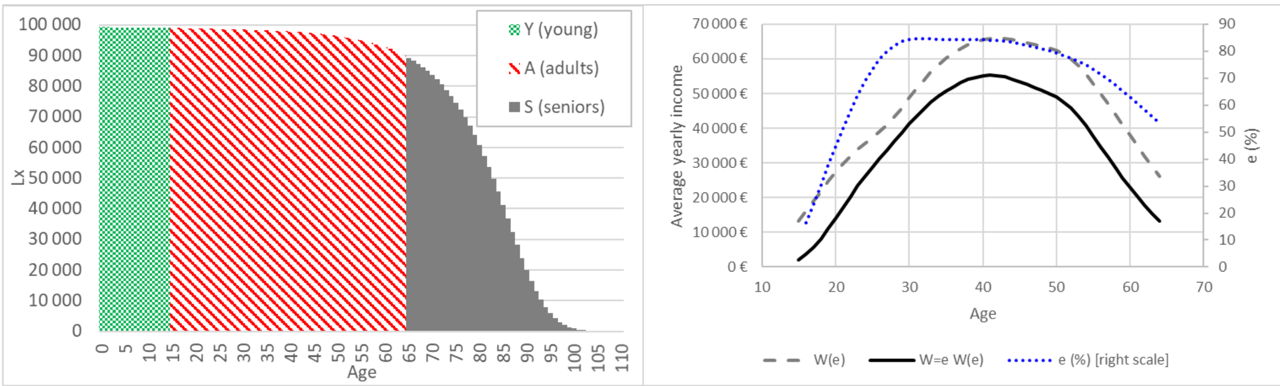
Table 1: List and explanations of the symbols used in this paper

Symbol	**U. M.**	**Description**
A	People	Number of adults in the stationary population
B	€	Child benefit
c	%	Contribution rate (*C/W*)
C_x, *C*	€	Average (age-specific or general) contribution per adult
d	Years	Difference between two average ages: μ_c and μ_b ($d = \mu_c - \mu_b$)
D	Years	Difference between two average ages: μ_p and μ_c ($D = \mu_p - \mu_c$)
e_x, *e*	%	(Age-specific or general) Employment rate (*E/A*)
E	People	*Employed* (*Ae*)
Gdp	€	Gross domestic product
Ipd	€	Implicit pension debt (in four different versions: 0, 1, 2, 3)
l_x	People	Number of individuals aged x in the stationary population (continuous case)
N_x, *N*	€	Average (age-specific or general) net labour income per adult. $N_x = W_x \cdot (1-c)$; $N = W \cdot (1-c)$
P_x, *P*	€	Average (age-specific or general) *P* benefit
Qcg	€	Quasi capital gains (and losses) = *PSD-BYd*
S	People	Number of older adults, or seniors, in the stationary population
T	People	Total stationary population (or total years of life lived)
W_x, *W* (W_e)	€	Average (age-specific or general) labour earning per adult (W_e = Average labour earning per employed person). $W = e \cdot W_e$
x	Years	Age
y	€	Average income
Y	People	Number of young persons in the stationary population
α, β	Years	Threshold ages marking the start (α) and the end (β) of adulthood (β=retirement age)
μ_c, μ_p, μ_b,	Years	Average year at transfers (*c*=paying contributions; *p*=receiving pension; *b*=receiving child benefits)

U. M.=Unit of measurement

A critical variable in this analysis is the population's age distribution. The left panel of Figure 1 illustrates the age structure of a stationary population – a scenario central to this paper – though real-world age pyramids are invariably more irregular, and sometimes considerably more. In this figure, shading distinguishes the three groups: *Y*, *A* and *S*.

Figure 1: Age segmentation of the survivorship curve (l_x) from a standard life table (left) and illustrative age-specific employment rates and labour earnings (right)



Notes: Stationary population with a life expectancy of 80.3 years (approximately the OECD average in 2023). Threshold ages α and β (here 15 years and 65 years, respectively) are used here to delineate the young, adult and older cohorts. Age-specific employment rates and average earnings are drawn from OECD data for 2022. Originally provided in quinquennial age groups, annual profiles were interpolated via cubic splines at the midpoint of each five-year interval. In the right panel, $W_x = e_x \cdot W_{e,x}$, where *e* represents the employment rate, W_e the average wage of the employed and *W* the average wage of the adults. All three variables are age specific (*x*=age).
Source: OECD Data Explorer.

The right-hand panel of Figure 1 presents three illustrative age-specific series for age *x*. The product of the employment rate e_x (E_x/A_x, plotted on the right-hand axis) and the average labour earnings $W_{e,x}$ yields the adults' labour earnings profile W_x ($=e_x \cdot W_{e,x}$). This variable – rarely, if ever, employed in economic analyses – proves valuable for our purposes, because it encompasses the two aspects that matter in the labour market (employment and productivity, by age) and because relates directly to the population age distribution, and in particular to the adults in the population (De Santis, 2024). Note that W_x is invariably lower than $W_{e,x}$, as it incorporates non-employed adults whose labour income is zero.
Paygo pension systems attain financial equilibrium when inflows (contributions) precisely equal outflows (pensions). Let *c* denote the constant contribution rate; contributions are then

1) $$\text{Contributions} = AWc = c\int_{x=\alpha}^{\beta} E_x W_{e,x}\,dx = c\int_{x=\alpha}^{\beta} l_x W_x\,dx$$

while pension disbursements are

2) $$\text{Pension benefits} = SP = \int_{x=\beta}^{\omega} l_x P_x\,dx$$

Equilibrium requires:

3) $$AWc = SP$$

While to date no Paygo system fully guarantees exact balance (some performing better than others in this respect), in principle this equality can be achieved through various parameter adjustments. In the stationary scenario, where nothing ever changes, budget balance is particularly straightforward to implement. One of the simplest solution is to fix the contribution rate ex ante – for example, at *c*=20%, near the 2023 OECD average, and to use the values from Figure 1 (*A*, *W*, and *S*) to determine P, so that the equality in (3) holds. Table 2 presents the resulting figures.

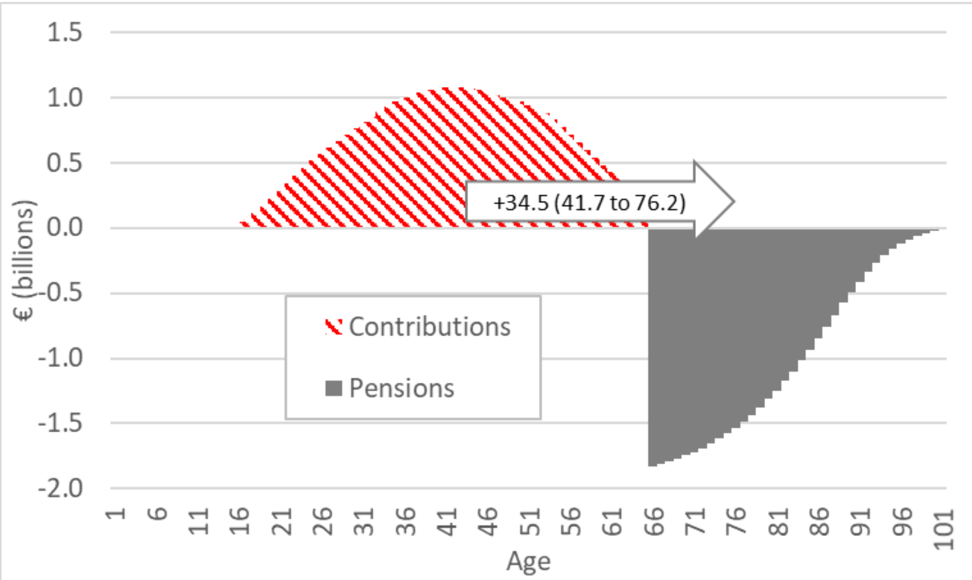
Table 2. Paygo variables in the stationary scenario of Figure 1 (with c=20%)

Category	**Variable**	**Symbol**	**Value**	**Source**
Inflows	Adults	A	≈4.8 million	See Figure 1
	Average adult wage	W	≈€36,000 per annum	See Figure 1
	Contribution rate	c	20%	Assumption
Outflows	Seniors	S	≈1.7 million	See Figure 1
	Average pension	P	≈€20,500 per annum	Dependent variable (formula 3)

Note: $W = e\, W_e$.

This scenario is depicted in the left-hand panel of Figure 2, where the red-hatched region denotes age-specific contributions in totality, and the grey-shaded region (plotted negatively) represents age-specific pension disbursements. When equality (3) holds, these two areas are congruent. Figure 2 further emphasises that the system effectively "transports resources forward" along both the age and time dimensions: the mean age of contribution payments (μ_c - see formula 7) is substantially lower than the mean age at which pension benefits are received (μ_p - see formula 5). In the current example, for instance, we get μ_c=34.5 years and μ_p=76.2 years. Their difference, D (slightly less than 35 years in this example), quantifies this temporal displacement. Naturally, varying threshold ages or altering the profiles of the underlying curves would affect the magnitude of D, though empirically such changes are modest. The significance of D for subsequent analysis will soon become apparent.

Figure 2: Age-specific aggregate monetary flows in the Paygo pension system based on Figure 1



Notes: Total transfers are displayed: contributions (red-hatched area), pensions (after age β), and, if present, child benefits, before age α. In this example, α=15 and β=65. The arrow denotes the temporal displacement: pension receipts lag contributions by approximately 34.5 years (average pension age = 76.2 years; average age of contribution payments = 41.7 years).

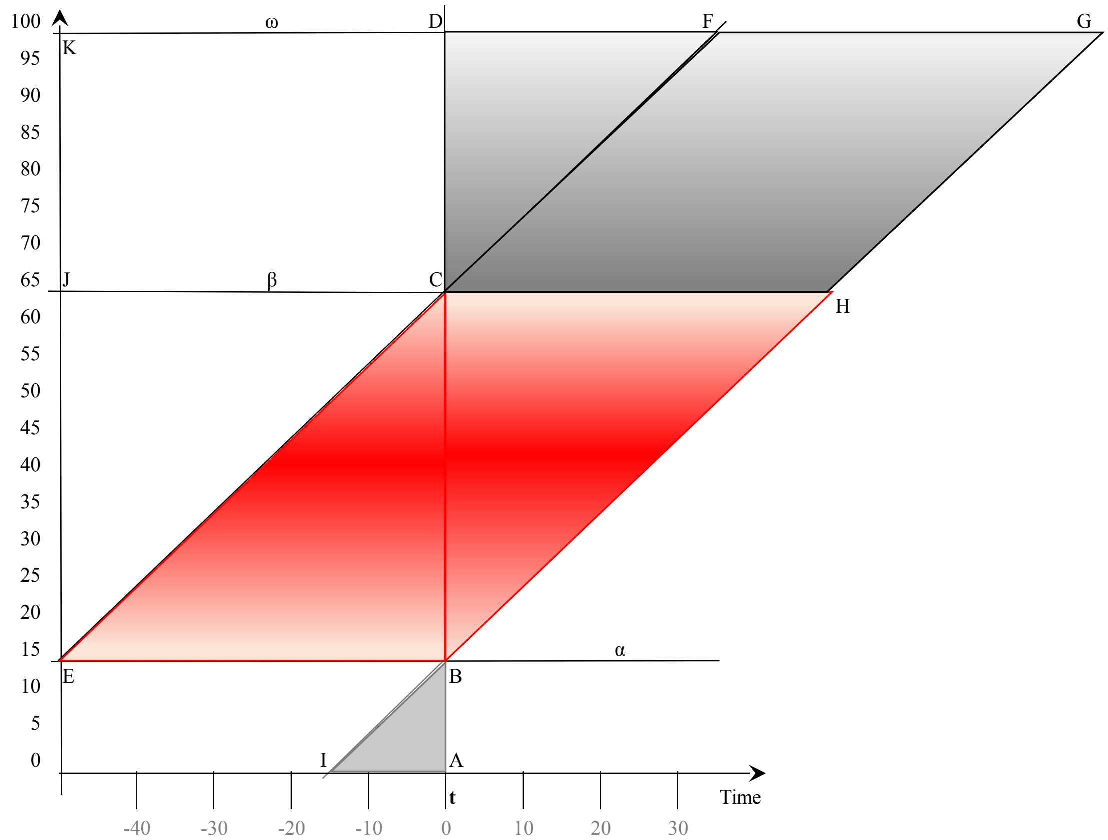
The left-hand curves from Figure 2 can be transposed onto a Lexis diagram – either cross-sectionally (vertical lines) or longitudinally (diagonal lines) – as depicted in Figure 3. It is crucial to recognise that these curves are now viewed "from above," so their height (third dimension in the graph) is not visible, and must be imagined. What appears as a line in this projection is actually the surface from Figure 2 (seen from above), and what looks like a surface is, in reality, a volume – a stack of identical curves, each resembling the left-hand panel of Figure 1. This situation is akin to surveying a varied terrain from a plane: though it may appear level, hills and valleys are present. In our context, the hills (in red) correspond to contributions received by the pension authority and the valleys (in grey) to pensions paid out.

Take, for example, the vertical line [BCD],[2] which captures the cross-sectional flows of contributions [BC] and pension payments [CD] for individuals alive at time *t* – characteristic of a Paygo system's snapshot. If the system is in financial equilibrium, these transfers sum to zero. Assuming financial equilibrium and stationarity, as we do here, all vertical lines reflect the same contribution–pension pattern as shown in Figure 1's left panel. Since in our scenario this holds for every vertical line (e.g. [EJK]), it also applies to the aggregated areas formed by these lines. For instance, the rectangle [EBDK] spans 50 years (from *t*−50 to *t*), encompassing 50 years of contributions [EBCJ] and 50 years of pension payments [JCDK]: their net sum is zero if the system has persistently maintained balance over this interval.

Paygo systems operate cross-sectionally yet produce significant longitudinal effects. Consider the diagonal line [ECF], which traces the payment history [EC] and subsequent pension receipts [CF] of the cohort born in year *t*−65. Under the stationary, balanced framework explored here, these cumulative transfers also net to zero; inspecting the system diagonally ([ECF]) yields an equivalent narrative to the vertical perspective ([BCD]). Of course, real-world systems deviate from this ideal – conditions change, both from the economic and the demographic point of view, and inflows rarely match outflows exactly.

[2] Brackets denote the geometrical elements from Figure 1.

Figure 3: Lexis diagram illustrating the four interpretations of implicit pension debt (Ipd) within Paygo pension schemes



Returning to our theoretical scenario, the parallelogram [EBGF] encapsulates the monetary transfers of the 50 cohorts born between $t-65$ and $t-15$ – namely, their contributions [EBHC] followed by pension receipts [CHGF]. Under the strict assumptions of stationarity and perpetual equilibrium, these two volumes are identical. In this theoretical construct, the triangles [EJC] and [BCH] are congruent; thus the square [EBCJ] represents the same aggregate contributions as the parallelogram [EBHC]. Similarly, in a stationary, balanced framework, the rectangle [JCDK] equals the parallelogram [CHGF] in pension disbursements.

We can now delineate four distinct interpretations of implicit pension debt (*Ipd*). The main conceptual difference between them lies in the degree of commitment to past pensions promises on part of current and future pension authorities, that is parliaments and governments. Pensions "vested" rights are strongest in the first case (Ipd_0), but progressively less and less certain in the other cases (Ipd_1 to Ipd_3). The last two (Ipd_2 and Ipd_3) rely on the assumption that current pension rules will not change, which is the less tenable, the more the analysis extends into the future.

2.1 The strictest interpretation: Ipd_0

Ipd_0 refers exclusively to the seniors at time t (segment [CD]) or, more precisely, to the current value of their future pension entitlements, represented by the triangular region [CFD]. Formally:

4) $$Ipd_0 = [\text{CFD}] = \int_{x=\beta}^{\omega} (x-\beta) P_x l_x \, dx$$

This measure corresponds to the implicit pension debt analysed, for instance, by Beltrametti and Della Valle (2011). Ipd_0 embodies pension obligations that are, in principle, hardest to reverse, as they accrue to retirees with fully vested rights – though in practice, even these may be subject to erosion, e.g. via inflation or special levies.

Note that the average age when these pensions are received (76.2 years in the example of Figure 2) is

5) $$\mu_p = \frac{\int_{x=\beta}^{\omega} x P_x l_x dx}{\int_{x=\beta}^{\omega} P_x l_x dx}$$

We will exploit this formula shortly.

2.2 Broadening the scope: Ipd_1

Ipd_1 encompasses all individuals who have already engaged with the pension system – namely, seniors and adults. Future contributions are excluded, but all those that took place in the past, and confer some rights, are counted. At time *t*, these cohorts correspond to segments [CD] (older adults, or seniors) and [BC] (adults). Seniors, as before, are entitled to Ipd_0 (triangle [CFD]), reflecting their vested pension rights. Likewise, were the scheme to be immediately terminated, adults [BC] would reasonably expect reimbursement of their past contributions, denoted by triangle [BCE]. Formally:

6) $$[BCE] = c \int_{x=\alpha}^{\beta} W_x (\beta - x) l_x dx$$

where W_x represents the average age-specific *adult* wage over the adult ages ($\alpha<x<\beta$), averaging W.
We may note that the average age at paying contributions is

7) $$\mu_c = \frac{\int_{x=\alpha}^{\beta} x W_x l_x dx}{\int_{x=\alpha}^{\beta} W_x l_x dx}$$

This formula, as was the case with formula (5), we will shortly prove useful.

Back to the main line of reasoning, Ipd_1 can be displayed on the Lexis diagram as the sum of two triangular regions (plus their non-visible heights):

8) $$Ipd_1 = [CFD] + [BCE] = \int_{x=\beta}^{\omega} (x-\beta) P_x l_x dx + c \int_{x=\alpha}^{\beta} (\beta - x) W_x l_x dx$$

Ipd_1 captures all the pension obligations rooted in past contributions – a concept discussed by Castellino (1985), in his measurement of the Italian *Ipd* in 1983. Ipd_1 is of course larger than Ipd_0, but it is also somewhat less certain, because the part of it that refers to the adult population will be claimed only in the distant future. Indeed, working-age adults must reach age β – up to (β−α) years from now, i.e. from year *t* – before they can claim their due, creating potential exposure to demographic, economic, or political shifts that could weaken the force of past promises and alter existing pension commitments.

2.3 Anticipating the foreseeable future: Ipd_2

Ipd_2 broadens Ipd_1 by incorporating both future contributions from current adults [BC] and their prospective pension receipts. On the Lexis diagram (Figure 3):

9) $$Ipd_2 = [CFD] + [CHGF] - [BCH]$$

In a stationary, balanced framework, longitudinal flows of pensions mirror contributions, hence:

10) $$[CHGF] = [BCE] + [BCH]$$

Substituting (10) into (9) shows that, in the stationary case, Ipd_2 equals Ipd_1 because

11) $$Ipd_2 = [CFD] + [BCE] + [BCH] - [BCH] = [CFD] + [BCE] = Ipd_1$$

In other cases, instead, and in real life in general, the equality does not hold. Usually, pension promises outstrip contributions, and Ipd_2 exceeds Ipd_1. Ipd_2 quantifies the hypothetical implicit debt were the current pension rules applied unchanged to everyone aged α and above at time t – excluding younger cohorts entirely.

2.4 To infinity and beyond: Ipd_3

Ipd_3 extrapolates Ipd_2 ad infinitum, presupposing that prevailing pension regulations will endure indefinitely. In the stationary, balanced scenario, all future cohorts replicate existing ones, yielding zero net social security balance – and thus:

12) $$Ipd_3 = Ipd_2 = Ipd_1$$

In practice, instead, pension commitments typically exceed contributions. In this case, Ipd_3 could diverge towards infinity. Yet actuarial discounting (e.g. $P_0 = \frac{P_t}{(1+i)^t}$) tempers this, making distant liabilities fiscally tractable – at least in present-value terms.

3. A new formula for Ipd_s (or Ipd_1 in the stationary, balanced case)

In what follows, I will refer to *Ipd* exclusively in the sense of Ipd_1 (henceforth Ipd_s) for three reasons:

1. Under stationarity and perpetual equilibrium, $Ipd_s = Ipd_1 = Ipd_2 = Ipd_3$.
2. Ipd_0 pertains solely to current retirees, whereas Ipd_1 encompasses obligations to all those who have already had some exchange with the pension system, including the adults who have already contributed, at least in part, but await pension benefits.
3. Even if future laws rescind pension promises, in part or in full, the implicit debt that has already been accumulated will not disappear: the resulting losses will be passed on to other stakeholders – contributors, taxpayers, future generations, or bondholders.

In the stationary, balanced model, Ipd_s admits a markedly simpler expression than given by formula (8). If we develop it, we get

13) $$Ipd_s = \int_{x=\beta}^{\omega} x\, P_x l_x dx - \beta \int_{x=\beta}^{\omega} P_x l_x dx + \beta c \int_{x=\alpha}^{\beta} W_x l_x dx - c \int_{x=\alpha}^{\beta} x\, W_x l_x dx$$

From (1), (2) and (3) we know that the central terms of this summation, those multiplied by β, are identical in the stationary and balanced case, because $\int_{x=\beta}^{\omega} P_x l_x dx$ represents total pension disbursements while $c \int_{x=\alpha}^{\beta} W_x l_x dx$ indicates total contributions (in any given year *t*, or for any birth cohort). As these two terms appear with opposing signs, they cancel out, and formula (13) simplifies to

14) $$Ipd_s = \int_{x=\beta}^{\omega} x\, P_x l_x dx - c \int_{x=\alpha}^{\beta} x W_x l_x dx$$

If we multiply and divide the first term by $\int_{x=\beta}^{\omega} P_x l_x dx$ (=*PS*) and the second by $c \int_{x=\alpha}^{\beta} W_x l_x dx$ (=*cWA*), remembering formulae (5) and (7), we obtain

$$Ipd_s = \mu_p \int_{x=\beta}^{\omega} P_x l_x dx - \mu_c c \int_{x=\alpha}^{\beta} W_x l_x dx = \mu_p PS - \mu_c PS = PS(\mu_p - \mu_c) = PSD$$

where, simplification is possible because *PS*=*cWA*, In short

15) $$Ipd_s = PSD$$

where *D* (=μ_p-μ_c) denotes the temporal displacement between average pension receipt and average contribution ages. Consequently, in equilibrium the implicit pension debt Ipd_s is *D* times the current pension load (*PS*). Note that *D* is typically large: in the vicinity of 35 in this example, but also more in general, as simulations with alternative values for the retirement age β indicate (not shown here).
To gauge Ipd_s relative to the gross domestic product (*Gdp*), two rough methods can be adopted. The first is to multiply the country's average pension-to-*Gdp* ratio by *D*. In OECD countries, for instance, this would result in 7.7% times 35, yielding about 2.7. This method, however, is incorrect, because it assumes stationarity and balance, neither of which applies to real life.
Alternatively, and preferably, indications cand be derived from

16) $$\frac{Ipd_s}{Gdp} = \frac{PDS}{yT} = \frac{P}{y}\frac{S}{T} D$$

where *y* is average income and *T* total years of life lived in a life table, or total stationary population. In the OECD case, for instance, in the years 2022–2023 (always assuming β=65 years), *P/y*≈43%, *S/T*≈21% and *D*≈35. Therefore Ipd_s/Gdp ≈ 3.2, which, by the way, is very close to Castellino's (1985) very worrying estimate of Ipd_1 for Italy in 1983.
Formula (15) also facilitates scenario analysis: if retirement age tracks longevity (stabilising the ratio *S*/*T* – a sort of long-term average for the share of seniors in the population, as shown by De Santis and Salinari 2023, 2024), and pension generosity (*P/y*) remains steady, Ipd_s/Gdp approximates a long-term norm, declining only during demographic upswings and rising in downturns.

4. From Ipd_s to quasi capital gains and losses, and vice versa.

Quasi capital gains (and, recently, losses), or *Qcg*, are generally presented in the following form (Lee 1980)

17) $$Qcg = PD\,\Delta S$$

In other words, an apparently magical "creation of resources" is made possible by the mere increase in the number of pensioners (ΔS). *Qcg* are large, because the increase in the number of potential beneficiaries ΔS is multiplied by the average value of pensions *P* and further multiplied by *D*, the difference between the two average ages, at receiving pension benefits and at paying contributions, which, as said, happens to be large, in the vicinity of 35. Incidentally, it is precisely this mechanisms, apparently creating resources out of nothing, that has permitted (or perhaps induced), several pension systems around the world to be excessively generous in their maturation phase, that is to promise more in pensions than what they declared they would collect in contributions, through various provisions, such as early retirement, non-actuarially equitable calculation formulae, etc. In this respect, formula (15) helps to clarify a few things.
First, quasi capital gains may derive not only from an increase in the number of seniors (as in formula 17 - admittedly, the most relevant), but also from an increase in the value of pensions (ΔP)

18) $$Qcg_{\Delta P} = \Delta P\, DS$$

or an increase in the age difference (ΔD), between receiving and paying pension transfers

19) $$Qcg_{\Delta D} = P\,\Delta D\, S$$

Second, all quasi capital gains correspond to an increase in *Ipd* (and quasi capital losses to its decrease), on a one-to-one basis. Indeed, the very creations of Ipd_s can be thought of as a series of successive increases in *P*, *D* and *S*, which have built up during the maturation phase of the system, and produced (or be on the way of producing) the final result. This perspective, by the way, permits to assess the monetary value of the frequently evoked "gift" made to the first beneficiaries of Paygo pension payments (e.g. Barr 2002, Feldstein 1974, Keyfitz 1985), and justifies the allegation that Paygo pension systems sinisterly resemble a Ponzi scheme.[3]
While quasi capital gains (*Qcg*) were enthusiastically received – if often poorly understood – during the maturation phase of Paygo pension systems, particularly by policymakers and their electorates (less so by scholars), they are now a cause for concern. The demographic conditions that once fuelled these gains – chiefly sustained population growth – have either stalled or are poised to reverse. As a result, the focus today is on how, if at all, the impact of demographic decline on the sustainability of Paygo pension systems can be mitigated.
Unfortunately, the only effective strategy to reduce Ipd_s – and thereby contain the risk of major future quasi capital losses – is to accept quasi capital losses in the present, i.e. to transform the implicit pension debt into an explicit one. Put differently, consider a simplified case in which we start from a suboptimal stationary and balanced scenario, where the prevailing level of Ipd_s is deemed excessive. All conceivable transitions to a more desirable stationary and balanced state –

[3] A Ponzi scheme is a type of fraud where money from new investors is used to pay returns to earlier investors, rather than from profit earned by the operation of a legitimate business. It creates the illusion of a profitable enterprise, but it eventually collapses when there isn't enough new money to pay existing investors.

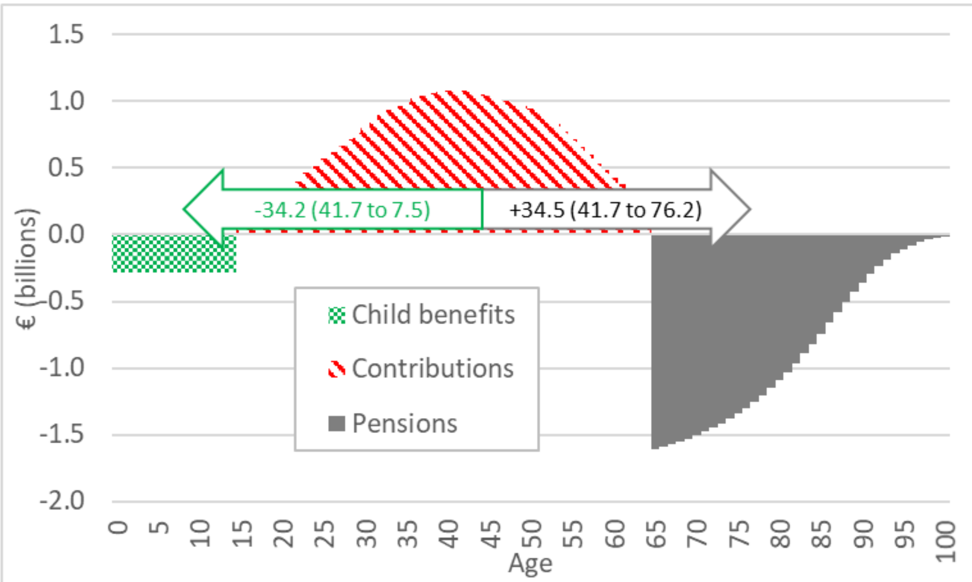
characterised by a lower Ipd_s – entail a transition cost, which corresponds exactly to the required reduction in Ipd_s, i.e. $\Delta(Ipd_s)$. There exists no costless, or just cheaper, pathway between the two equilibria.

One obvious approach to easing this burden is to engineer a gradual transition from the current, allegedly sub-optimal, steady state to the improved one, thereby diluting the adjustment costs over time. However, protracted transition phases are not without drawbacks – political, economic, and intergenerational. In the real world, matters are further complicated by the fact that substantial costs are already looming, even in the absence of reform, due to the demographic contraction that is either imminent or already under way in several countries where Paygo pension systems are already operating.

5. From pensions to intergenerational transfer systems?

Despite the considerable political and economic challenges of undertaking structural reforms to Paygo pension systems – especially in an era of population ageing – future generations may wish to consider a more radical transformation: converting Paygo pensions into broader intergenerational transfer systems. Such systems would reallocate resources not only to the elderly (those aged β and above) but also to the young (those below age α), albeit not necessarily in equal measure.

Figure 4: Age-specific aggregate monetary flows in the Paygo pension system based on Figure 1, with child benefits



Notes: Total transfers are displayed: contributions (red-hatched area), pensions (after age β), and, if present, child benefits, before age α. In this example, α=15 and β=65. Arrows denote temporal displacement: pension receipts lag contributions by approximately 34.5 years (average pension age = 76.2 years), while child benefits (when included) *precede* contributions by about 34.2 years (average benefit age = 7.5 years). Average age of contribution payments = 41.7 years.

This scenario is schematically illustrated in Figure 4. Although other solutions could be envisioned, the contribution rate is assumed to remain constant at 20%, for simplicity, while pension benefits are reduced (from approximately €20.5k to €18k per year, a decline of roughly 12%). This creates fiscal space for the introduction of a child benefit amounting to around €3k per year per child (up to age α=15). In this configuration, resources are no longer exclusively transferred "upwards" (to

seniors), but also "downwards" (to children), thereby partially counterbalancing the traditional intergenerational flow.
In this case, in looking at the Lexis diagram of Figure 3 child-related transfers must be included, and these are those already received by the young cohorts [AB] at time *t*, denoted as [ABI]. These transfers should be repaid by the beneficiaries, in case of a sudden discontinuation of the transfer system. With appropriate adjustments – not detailed here – the *Ipd* formula originally presented as equation (15) becomes:

20) $$Ipd_{sy} = PDS - BdY$$

Where *B* represents the annual child benefit, *Y* denotes the number of children (i.e. individuals aged below α), and $d=\mu_c-\mu_b$ is the difference between the average age at which contributions are paid (μ_c) and the average age at which child benefits are received (μ_b).
An illustrative numerical example is provided in Table 3. In brief, the transition from Ipd_s to Ipd_{sy} results in a 25% reduction in implicit pension debt. This reduction is attributable in part (~12%) to the lower pension benefits, and in part (~13%) to the introduction of child benefits. The latter contribute negatively to the *Ipd* measure, since they represent downward transfers in the age (and time) dimension.

Table 3 – Implicit pension debt in the stationary and balanced case, with and without child benefits. Illustrative values based on author's assumptions

Symbol	**Unit of measure**		$\boldsymbol{Ipd_s}$		$\boldsymbol{Ipd_{sy}}$		**Index number**
D	years		34.5		34.5		100
P	(000 €/year)		20.5		18		88
S	mil		1.7		1.7		100
$\boldsymbol{Ipd_s}$	(000 €/year)		***1202***		***1056***		***88***
d	years				-34.2		
B	(000 €/year)				2.9		
Y	mil				1.5		
$\boldsymbol{Ipd_{sy}}$	(000 €/year)				***-149***		
Ipd	(000 €/year)		**1202**		**907**		**75**

Note: See Figure 1 and accompanying text. Assumptions: contribution rate *c* = 20%; average pension *P* = €18,000/year; average child benefit *B* ≈ €2,900/year. *W = e We*. Author's calculations.

Although the figures in Table 3 are purely illustrative, the overall message has general validity: any reduction in the average pension amount, offset by the introduction of child benefits, would have a dual effect on Ipd_{sy} – namely, a lower "traditional" Ipd_s component and the deduction of the *BdY* term, which has a negative sign, because this transfer of resources is backward.

6. Discussion and conclusions

This article has developed a compact and transparent analytical framework for understanding the dynamics of implicit pension debt (*Ipd*) within Pay-as-you-go (Paygo) systems. Its central contribution lies in the formulation of *Ipd* in stationary and balanced conditions as the product of three key parameters:

$$Ipd_s = PDS$$

where P is the average annual pension, D is the average age difference between when contributions are paid and pensions are received, and S is the number of retirees. Although the literature has long recognised that Paygo systems implicitly generate liabilities (e.g. Castellino 1985; Holzmann et al. 2001; Beltrametti and Della Valle 2011), the formulation proposed here appears to be original in its algebraic simplicity and interpretative clarity.

This formulation not only helps disentangle the determinants of *Ipd*, but also provides a coherent basis for analysing quasi capital gains and losses, which emerge when P, D or S change over time (Lee 1980). These gains and losses are not fictional or residual: they correspond to real shifts in the demographic and institutional structure of Paygo systems. In fact, during the growth phase of these systems – when the number of contributors and retirees increased, and average benefits were gradually scaled up – quasi capital gains often went unnoticed or were mistaken for genuine "resource creation", or at least fiscal space. In fact, they derived merely from the accumulation of liabilities, the magnitude of which can now be better assessed, thanks to formula (15). Today, with population ageing and potential decline on the horizon, the risk of large quasi capital losses looms larger, and calls for closer scrutiny.

To mitigate the long-term vulnerability of Paygo systems to demographic contraction, a generalisation of the *Ipd* framework can be considered, to incorporate intergenerational transfers not only to the elderly but also to the young. This extension is represented by the enlarged formula:

$$Ipd_{sy} = PDS - BdY$$

where B is the average annual child benefit, Y the number of young individuals (below age α), and d the average age difference between contribution payment and benefit receipt by the young. The introduction of downward transfers to younger cohorts – modeled here as child benefits – contributes negatively to the implicit debt, thus reducing the overall liability of the system. This result is both intuitive and policy-relevant: a reallocation of resources from elderly-only transfers to a broader intergenerational distribution can yield more balanced outcomes without necessarily raising contribution rates or destabilising public finances. The illustrative calculations presented in this article show that any reduction in pension levels, offset by the introduction of moderate child benefits, roughly doubles its effects. Incidentally, several authors convincingly argue that pension systems deprive children of their economic utility, and therefore contribute to depress fertility (e.g. Rossi and Godard 2022; Sánchez-Barricarte 2017). According to some scholars, a possible solution would consist in linking future pension benefits to the number and productivity of the pensioners' adult children (Cigno and Werding 2007; Demeny 2015; Regős 2015). The introduction of direct transfers towards children would act in the same spirit and in the same direction, although of course its effectiveness remains to be tested.

In the broader policy debate, these results reinforce the need to treat *Ipd* as a meaningful and measurable indicator of fiscal and demographic sustainability – not as an accounting fiction. As emphasised by Settergren and Mikula (2006), understanding the present value of future obligations is essential for rational pension design, especially in systems where liabilities are not prefunded. The framework proposed here facilitates this understanding, and offers a tractable tool for policy analysis in an area often clouded by political opacity and technical complexity.

In conclusion, the compact expressions for *Ipd* and its components presented in this article provide a clear and flexible framework for rethinking Paygo pensions in a context of demographic change. They reveal the mechanisms through which implicit debts are created, modified, and potentially reduced – whether by altering benefit levels, recalibrating age structures, or expanding the scope of transfers. In so doing, they offer not just a diagnostic tool, but also a conceptual foundation for building more resilient and intergenerationally balanced social protection systems.

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