The Formal Demography of Populations with Declining Fertility

Gustav Feichtinger Roland Rau Andreas Novak Stefan Wrzaczek

October 14, 2025

Extended Abstract prepared for the European Population Conference 2026

Abstract

Falling fertility and population decline do not seem like major news. Fertility decline has become a global phenomenon since at least the 1970s, when fertility rates started dropping rapidly in many middle- and lower-income countries, including China. The ongoing fertility transition is perceived as a one-end road: once fertility decline starts, it is likely to continue—driven by expanding education, urbanization, development, access to contraception, and changing reproductive preferences—until a country reaches a low fertility level. Persistent low fertility rates and accelerating fertility declines in many world regions have fostered the fear of falling numbers of births and population declines that might threaten economic growth, prosperity, and social security systems. In the present contribution we don't care about all these fears but apply formal demographic modelling to systematically explore the short- and long-term consequences of constantly falling fertility.

Among the important insights we obtain are the following:

- The peak of births is reached half a generation length before replacement level fertility.
- A resulting 'window of opportunity' occurs just after some 30 years - the so-called *demographic dividend*. The size of this window depends specially on the key parameters of fertility decline.
- In a shrinking population the population momentum has a pronounced impact on the size and the age structure of the population.

Since nobody expects fertility to increase immediately to replacement level (i.e. by a jump), the question arises how birth rates should gradually grow such that there is a satisfactory (i.e. optimal) trade-off between the size and the age structure of the resulting population. Frejka (1973) and Masnick (1973, p. 339) referred quite early to the involved *optimisation problem* arising in population policy. Although they formulated the problem

for growing populations, it clearly does exist for declining populations as well. Speed and extent of the fertility increase should be chosen to avoid both, a further shrinkage and intensified fluctuations in the age composition.

Two remarks are important in this context:

First: Formal demography relies on modelling, for which simplifications are essential. Due to the fact that *analytical* statements are possible, logically, they don't describe *real* processes. Instead of this, we get 'if - then' results.

Second: How relevant is the model approach with respect to demographic reality? One of the fathers of demography gives the right answer to this question by his speedometer-allegory (compare Coale, 1972).

In what follows we exemplify these ideas by explaining the optimisation problem mentioned above as a last point. We select it, since we are convinced that it can be seen as backbone of a theory of population policy (which is in our opinion still missing).

Using the McKendrick equation describing the dynamics of an age structured female population with the rate of fertility change as control variable and the trade-off between the size of the population and its fluctuations in the age structure as objective functional, a variant of PONTRYAGIN's *maximum principle* is applied to solve the intertemporal decision problem a policymaker is facing. It turns out that the qualitative insights into the structure of the optimal timing of the fertility dynamics are quite robust. Note further that interesting analytical results can be obtained for assuming concentrated vitality, i.e. rectangular survival as well as a unique (mean) age at birth.

Finally, some extensions are discussed such as the optimal age of immigration in sub-replacement populations (maximising the support ratio) as well as applications in manpower planning in hierarchical organisations such as learned societies.